D 73763 **(Pages : 4)** Name..

Reg. No.....................................

THIRD SEMESTER (CUCBCSS—UG) [SPECIAL] DEGREE EXAMINATION NOVEMBER 2019

Statistics

STS 3C 03—STATISTICAL INFERENCE

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word**.
Each question carries 1 *mark.*

Name the following :

1. Probability distribution of a statistic.
2. Probability distribution of the square of a random sample taken from a standard normal population.
3. Name of the estimator 't' for a parameter θ, where $E(t) = \theta$.

Fill up the blanks :

4. The probability of the value of a parameter to fall within a confidence interval is called the ———————— of that interval.
5. If the mean of a Chi-square random variable is 4, its variance is ————————.
6. The hypothesis is to be tested is called ———————— hypothesis.
7. ———————— of any statistic is called its standard error.

Write True *or* False :

8. It T is a consistent estimator of θ, then $V(T)$ increases as sample size increases.
9. Method of moment is one of the methods to identify an estimator for a parameter.
10. In a small sample test of the mean of a normal population, the test statistic follows Chi-square distribution.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in one sentence each.*
Each one carries 2 marks.

11. Define Statistic.

12. Identify the probability distribution of $\sum_{i=1}^{n}\left[\frac{x_i - 10}{3}\right]^2$ where x_i's are random samples taken from normal distribution with mean 10 and variance 9.

13. Define F-distribution.

14. Define a sufficient estimator.

15. Define interval estimation.

16. Define simple and composite hypothesis.

17. Define critical region.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each one carries 4 marks.

18. State and prove the additive property of chi square distribution.

19. For a Poisson distribution with parameter λ, show that sample mean is an unbiased estimator of λ.

20. Show that the sample mean is a M L E of population mean when sample is taken from normal population.

21. Obtain the mean of t, where t follows t-distribution with n degrees of freedom.

22. Define type I and type II errors in testing of hypothesis.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each one carries 6 *marks.*

23. It X_1, X_2 and X_3 is a random sample taken independently from N(0, 1) population, identify the probability distributions of (i) $X_1^2 + X_2^2$; (ii) $\frac{X_3}{\sqrt{\frac{X_1^2 + X_2^2}{2}}}$;and (iii) $\frac{X_1^2}{X_2^2}$.

24. If X is distributed as $f(x) = \frac{1}{\theta}, 0 < x < \theta,$ show that $-2\log_e\left(\frac{x}{\theta}\right)$ follows Chi-square distribution with 2 degrees of freedom.

25. Find the M.L.E. of μ and σ, using the random sample $x_1, x_2, \ldots, x_n$ taken from the normal population $N(\mu, \sigma^2)$.

26. Derive confidence interval for the variance of a normal population when population mean is unknown.

27. To test $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 3,$ based on a random sample of size one taken from a population following rectangular distribution over $[0, \theta]$. Find the size and power of the test if the critical region is $x > 1.5$.

28. Explain the interrelation between Chi-square, *t* and F distributions.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each one carries 10 *marks.*

29. Derive the sampling distribution of the mean of random sample $x_1, x_2, \ldots, x_n$ taken from a normal population $N(\mu, \sigma^2)$.

30. Derive any *two* statistics following *t*-distribution.

Turn over

31. (i) Explain the method of testing equality of population proportions, when large samples are taken independently from two populations.

(ii) A random sample of 200 items produced by machine A and 300 bolts produced by machine B gave 14 and 20 defective items respectively. Test the hypothesis that the machine A shows higher quality of performance at 5 % level of significance ?

32. (i) Explain Chi-square test of independence of attributes.

(ii) A group of randomly selected individuals are classified according to their English language proficiency and numerical ability :

Proficiency in English	Numerical ability		Total
	Yes	No	
Yes	50	26	76
No	20	30	50
	70	56	126

Test whether the two attributes are dependent.

(2 × 10 = 20 marks)

D 12052 (Pages : 3) Name.....................................

Reg. No.................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY THEORY

(2019 - 2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of calculator and Statistical table are permitted.

Section A

Answer atleast **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall ceiling 24.

1. Define : (i) Random experiment ; and (ii) Sample space.

2. If A and B are mutually exclusive events, obtain the value of $P(A \cap B)$.

3. What are the axioms on probability ?

4. Given $P(A) = 0.7$, $P(B) = 0.6$, $P(A \cup B) = 0.9$, find $P(A / B)$.

5. For two independent events A and B with respective probabilities 0.5 and 0.4, find the probabilities of (i) $P(A \cap B)$; (ii) $P(B / A)$.

6. State the multiplication theorem on probability for two events A and B.

7. If the p.m.f. of X, $f(x) = 0.25$, for $x = 1, 2, 3$ and 4 ; obtain E(X).

8. Find $P(2 \leq X < 5)$ if the p.m.f., of X, $f(x) = (4x - 3)/45$, $x = 1, 2, 3, 4, 5$.

9. 2 balls are randomly taken at a time from a box containing 5 black and 3 white balls. Find the probability of getting 2 black balls.

10. Write any two properties of standard normal distribution.

11. If X follow Bernoulli distribution with parameter $p = 0.4$, obtain E(X).

12. Define discrete uniform distribution.

(8 × 3 = 24 marks)

Turn over

Section B

Answer atleast **five** *questions.*

Each question carries 5 marks.

All questions can be attended.

Overall ceiling 25.

13. The probabilities of the events A, B and $A \cap B$ are respectively 0.6, 0.5 and 0.3. Calculate the probabilities of (i) A^c ; (ii) $A^c \cup B^c$; (iii) $A^c \cap B$; and (iv) $A \cap B^c$.

14. State the addition theorem on probability for two events A and B. For two independents events A and B, calculate $P(A \cup B)$, if $P(A) = 0.4$, $P(A \cap B) = 0.2$.

15. Write the sample space of a random experiment of tossing of three fair coins simultaneously. Identify the probabilities of getting : (i) at least one head ; (ii) at most two heads ; and (iii) no heads.

16. For a discrete random variable X, define : (i) probability mass function ; and (ii) distribution function. Also state any two properties of probability mass function.

17. Define mathematical expectation. Find E(X) if the p.m.f. of X is $f(x) = \frac{x^2}{14}$, for $x = 1, 2, 3$.

18. An experiment with two possible results success and failure with the probability of success 0.4 is repeated independently for 20 times. If X is the total number of successes obtained. Identify the probability mass function of X. Calculated the probabilities of getting : (i) no successes ; and (ii) no failures out of these 20 trials.

19. Define Poisson distribution and state its properties.

(5 × 5 = 25 marks)

D 12051 (Pages : 2) Name.......................................

Reg. No..................................

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 02—PROBABILITY DISTRIBUTIONS AND PARAMETRIC TESTS

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. If X denotes total number of heads when an unbiased coin is tossed 20 times. Obtain the mean and variance of X.
2. If X is a Poisson random variable with parameter 5, find P $(X < 1)$.
3. Write the p.d.f, mean and variance of normal distribution.
4. Find P $(Z < 2)$ and P $(Z > -1)$, where Z follow standard normal distribution.
5. Define sampling frame.
6. Define simple random sample.
7. Define judgment sampling.
8. Define null and alternate hypothesis.
9. Define significance level.
10. Define type II error.
11. Calculate the value of test statistic in testing the mean $\mu = 30$ of a population using a random sample of size 225 with the mean and variance, 32 and 16 respectively.
12. Write the test statistic and its probability distribution used in a small sample test of equality of variances of two normal populations.

$(8 \times 3 = 24 \text{ marks})$

Section B

Answer at least **five** *questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Given the mean and variance of a binomial distribution as 6 and 4.2 respectively. Find (i) P (X= 10) ; (ii) P (0 < X < 3).

14. Variance of a Poisson random variable X is 3. Write the p.m.f. of X. Obtain the mean of X and P (X > 1).

15. The completion time X of a certain job is normally distributed with mean 8 hours and with variance 4 hours. What are the probabilities to complete the job (i) within 6 hours ; (ii) with more than 12 hours.

16. Explain the method of stratified sampling.

17. A random sample of size 100 is found to have mean 342. Could it be reasonably regarded as a sample from a population with mean 330 ,where the population standard deviation is 66 at a 5% significance level?

18. A random sample of 120 villages taken from a certain district shows the average population per village as 480 with a standard deviation 50. Another random sample of 160 villages from neighboring district shows the average population per village as 440 with a standard deviation 60. Test whether the average population per village for these two districts differs significantly at 5% level of significance.

19. A random sample of size 12 drawn from a normal population is 34, 23, 46, 57, 36, 28, 41, 45, 22, 30, 44, 32. Test whether the population variance is 6 at $\alpha = 0.05$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Explain the method of testing of population proportion, using large sample.

    (ii) A random sample of 500 items produced by a machine show 36 defectives. Test the claim the machine is producing only 5% of defectives at 5% level of significance.

21. Explain (i) small sample test for equality of means of two normal populations with same but unknown variances ; (ii) Paired t-test.

(1 × 11 = 11 marks)

D 12050-A **(Pages : 4)** **Name..................................**

Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019–2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(Multiple Choice Questions for SDE Candidates)

1. For a binomial distribution :

   (A) Mean = Variance. (B) Mean < Variance.

   (C) Mean > Variance. (D) Changes with parameters.

2. Binomial distribution is negatively skewed when :

   (A) $p < q$. (B) $p > q$.

   (C) $p = q$. (D) $p \neq q$.

3. A discrete random variable takes 10 values with equal probability. Then its distribution is :

   (A) Binomial distribution. (B) Poisson distribution.

   (C) Bernoulli distribution. (D) Discrete uniform distribution.

4. The mean and variance of a binomial random variable is 6 and 4 respectively. Then its distribution is :

   (A) $B\left(18, \frac{1}{3}\right)$. (B) $B\left(18, \frac{2}{3}\right)$.

   (C) $B\left(18, \frac{1}{2}\right)$. (D) None of these.

5. If $X \sim B(n, p)$, the distribution of $n - X$ is :

   (A) Binomial. (B) Poisson.

   (C) Negative binomial. (D) None of these.

6. Which of the following is not a condition for Poisson approximation of binomial ?

   (A) n is large. (B) p is small.

   (C) $np = \lambda$ is small. (D) $np = \lambda$ is a constant.

7. Which of the following distribution have lack of memory property ?

   (A) Geometric. (B) Exponential.

   (C) Both (A) and (B). (D) Neither (A) nor (B).

8. The distribution of the sum of n independent geometric random variable is :

(A) Binomial. (B) Poisson.

(C) Negative binomial. (D) None of the above.

9. If X and Y are independent uniform random variables over [0, 1], then $P(X < Y)$ is :

(A) 0.1. (B) 0.2.

(C) 0.5. (D) 0.7.

10. In a normal distribution QD : MD : SD is equal to :

(A) 10 : 15 : 12. (B) 12 : 10 : 15.

(C) 10 : 12 : 15. (D) 12 : 15 : 10.

11. Under a normal curve 95.45% of the items lies between :

(A) $\mu - \sigma$ and $\mu + \sigma$. (B) $\mu - 2\sigma$ and $\mu + 2\sigma$.

(C) $\mu - 3\sigma$ and $\mu + 3\sigma$. (D) $\mu - 4\sigma$ and $\mu + 4\sigma$.

12. The area, between 2.3 and 4.7 under a normal curve is 0.2180. Then $P(2.3 < X < 4.7)$ is :

(A) 0.4360. (B) 0.0640.

(C) 0.2820. (D) 0.2180.

13. The mode of convergence in central limit theorem is :

(A) Convergence in distribution. (B) Convergence in probability.

(C) Convergence almost surely. (D) None of the above.

14. Chebyshev's inequality does not give any significant result for :

(A) $k < 1$. (B) $k = 1$.

(C) $k > 1$. (D) None of the above.

15. Let N = 12. If a systematic sample of size 4 is taken, then which of the following is a possible sample :

(A) (1, 4, 7, 10). (B) (2, 5, 8, 11).

(C) (3, 6, 9, 12). (D) All the above.

16. A sample consists of :

(A) All the units of the population.
(B) 50% of the population.
(C) 25% of the population.
(D) Any fraction of the population.

17. Sampling error :

(A) Increases with sample size.
(B) Decreases with sample size.
(C) First increases and then decreases.
(D) First decreases and then increases.

18. Difference between a statistic and the parameter is called :

(A) Sampling error.
(B) Non-sampling error.
(C) Standard error.
(D) None of these.

19. Sampling distribution is the probability distribution of :

(A) Parameter.
(B) Statistic.
(C) Variable.
(D) Sample.

20. Mean of Chi-Square distribution with n degrees of freedom is :

(A) n.
(B) $2n$.
(C) $\frac{n}{2}$.
(D) n^2.

D 12050 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3C 03—PROBABILITY DISTRIBUTIONS AND SAMPLING THEORY

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted.

Section A (Short Answer Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Find the mean of a uniform random variable with possible values 1,2,3,4 and 5.
2. Define negative binomial distribution.
3. Obtain the m.g.f. of exponential distribution.
4. If X follow normal distribution with mean 10 and variance 9, find (i) $P(X > 13)$; (ii) $P(7 < X < 13)$.
5. Define parameter and statistic.
6. Define convergence in distribution.
7. State Central Limit theorem.
8. Define the terms (i) population ; (ii) sampling frame.
9. Define simple random sampling.
10. Find the probability that the sample mean of a random sample of size 16 taken from a normal population with mean 2 and variance 4 is less than 1.
11. If X and Y are independent standard normal random variables, identify the probability distribution of $\left[\frac{X-Y}{X+Y}\right]^2$.
12. Define t-distribution.

(8 × 3 = 24 marks)

Turn over

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. If the $(r-1)^{th}$, r^{th}, and $(r+1)^{th}$ central moments of X following binomial distribution with parameters n and p are, μ_{r-1}, μ_{r-1} and μ_r, show that $\mu_{r+1} = pq\left[nr\,\mu_{r-1} + \frac{d}{dp}\mu_r\right]$.

14. State and prove lack of memory property of exponential distribution.

15. State and prove Chebyshev's inequality.

16. Examine whether Weak Law of Large Numbers hold for the sequence of random variable $\{X_i\}$, where $P\left(X_i = \pm\sqrt{2i-1}\right) = \frac{1}{2}$.

17. State and prove Bernoulli's law of large numbers.

18. Explain cluster sampling.

19. A random sample of size 10 is taken from a normal population with mean 10 and unknown variance. If the sample variance are is 18.23, find the probabilities of the sample mean (i) less than 9 ; (ii) greater than 11.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (i) Show that odd order central moments of X following $N(\mu, \sigma^2)$ are zeroes.

(ii) Prove that the mean deviation about the mean of X following $N(\mu, \sigma^2)$ is $\sqrt{\frac{2}{\pi}}\,\sigma$.

21. (i) Define Chi-square distribution. Obtain the m.g.f., of X following $\chi^2_{(n)}$.

(ii) State and prove the additive property of Chi-square distribution.

(1 × 11 = 11 marks)

D 12049 (Pages : 3) Name.......................................

Reg. No..................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Statistics

STA 3B 03—STATISTICAL ESTIMATION

(2019–2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Use of calculator and Statistical table are permitted.

Section A (Short Answer Type Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find a and b, if the mean and variance of X following continuous uniform distribution over $[a, b]$ are respectively 5 and 3.
2. Write the mean and p.d.f., of X following exponential distribution with variance 0.25.
3. Define gamma distributions with one and two parameters.
4. Define Cauchy distribution. State any two properties of this distribution.
5. If X is a random variables following normal distribution with means 8 and variance 9. Identify the probability distribution of $Z = 2X - 5$.
6. A random sample of size 9 is taken from a normal population with mean 6 and variance 9, Identify the probability that the sample variance is less than 5.07.
7. If X follow $N(5, 3^2)$, find (i) $P(X < 2)$; (ii) $P(X > 8)$.
8. If X and Y are independent standard normal random variables, name the probability distribution of (i) $X^2 + Y^2$; (ii) X^2 / Y^2.
9. Differentiate Estimator and estimate.
10. Define efficient estimator. A random sample of size 2 is taken from $N(\mu, \sigma^2)$. Which of the following is more efficient estimator for μ, (i) $\frac{x_1 + x_2}{2}$; (ii) $\frac{x_1 + 2x_2}{3}$.
11. Define Cramer-Rao Lower Bound.
12. Write any four properties of a maximum likelihood estimator.

13. Define the moment estimator.

14. Define (i) interval estimation (ii) confidence interval

15. Write 90% confidence interval for the mean of a normal population with variance 9 using a random sample of size 16 from this population having sample mean 8.

(10 × 3 = 30 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. If X is a random variable following rectangular distribution over $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, prove that Y = tan X follow standard Cauchy distribution.

17. The life length of an LED TV is an exponential random variable with a mean life 8 years. What is the probability that such a TV to last at least for 10 years ?

18. Show that the harmonic mean of X following beta distribution of first kind with parameters p and q is $\frac{p-1}{p+q-1}$.

19. If X follow $N(10,3^2)$, find a, such that P (X < a) = 0.76.

20. Derive the sampling distribution of the mean of a random sample of size n taken from $N(\mu,\sigma^2)$.

21. Define a consistent estimator. Prove that the sample mean of random sample of size n taken from a Poisson distribution with p.m.f $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2,\ldots$ is a consistent estimator of λ.

22. Obtain the MLE of θ for the population with probability distribution $f(x) = \frac{1}{2}e^{-|x-\theta|}, -\infty < x < \infty$ using a random sample of size n.

23. Derive a 90% confidence interval for the mean of a normal population $N(\mu,\sigma^2)$ with unknown population variance using a random sample of size 16.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (i) State and prove the lack of memory property of exponential distribution.

    (ii) If X follow uniform distribution over [0,1], prove that $Y = -2\log_e X$ follow exponential distribution.

25. (i) If X follow $N(\mu, \sigma^2)$, show that the mean, variance of X are respectively μ, σ^2.

    (ii) Prove that the odd order central moments of X following $N(\mu, \sigma^2)$ are zeroes.

26. Define chi square distribution. Obtain the m.g.f, of X following chi-square distribution with n degrees of freedom. Also state and prove the additive property of chi-square distribution.

27. (i) Derive a 100 $(1-\alpha)$% confidence interval for the proportion *p* of a binomial population based on a random sample of large size ; (ii) In a random sample of 500 readymade garments produced, 25 are found to be defective. Obtain a 95% confidence interval for the population proportion of defective items..

(2 × 10 = 20 marks)

D 11880 (Pages : 4) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Statistics

SG 3C 03—PROBABILITY

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word.**

Each question carries 1 mark.

Fill up the blanks :

1. The result of a random experiment is called as ————.
2. The probability of drawing any one spade card from a pack of cards is ————.
3. If three coins are tossed at a time, then the numbers of sample points in sample space are ————.
4. If X is a random variable and '*c*' is a constant, then $E\left(\frac{c}{X}\right) =$ ————.
5. Expectation of a random variable exists if ————.
6. The mode of the Normal distribution is ————.
7. The mean and variance of a Poisson distribution are ————.

Write True *or* False :

8. Two events are said to be mutually exclusive if both the events has same favourable cases.
9. The range of variation of the distribution function of a random variable is $-\infty$ to $+\infty$.
10. Continuous random variables always assume non-integer values.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in* **one sentence** *each.*
Each question carries 2 marks.

11. Define probability density function.

12. Define pair wise independence.

13. What are the limitations of axiomatic definition of probability ?

14. Given the distribution function $F(x) = 1 - e^{-x}, x \geq 0$. Find p.d.f.

15. Given $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{6}$, find $P\left(\frac{B}{A}\right)$.

16. State additional theorem on expectation for '*n*' random variables $x_1, x_2, \ldots\ldots x_n$.

17. If V (X) = 4 in binomial distribution (*n*, 1/3), find *n*.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 marks.

18. Find variance of Poisson distribution.

19. Show that probability of impossible event is zero.

20. Distinguish between pair wise independent and mutually independent events with examples.

21. Define distribution function and state its properties.

22. Define mathematical expectation of a random variable. Calculate E(X) if X has pdf

$f(x) = \frac{1}{4}, x = 1, 2, 3, 4$ and zero otherwise.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 marks.

23. (i) If A and B are independent, show that A and B^c are independent.

(ii) If A and B are independent show that A^c and B^c are independent.

24. A random variable has the following probability function :

X	-2	-1	0	1	2	3
P (X)	0.1	k	0.2	$2k$	0.3	k

(i) If $k = 0.4$, examine whether $p(x)$ is a p.d.f.

(ii) Find the value of k.

25. Describe the approximation of Poisson distribution to binomial distribution.

26. The mean and variance of a binomial distribution are 2.5 and 1.875 respectively. Obtain the binomial probability distribution. Find $P(x \leq 3)$.

27. If X is the member of points rolled with a balanced die, find the expected value of $2X^2 + 1$ and $\frac{X}{2}$.

28. Find the distribution function of a total number of heads obtained in 4 tosses of a balanced coin.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Three newspapers P1, P2 and P3 are published in a city. From a survey, it is estimated that 20% read P1, 16% read P2,14% read P3, 8% read P1 and P2, 5% read P1 and P3, 4% read P2 and P3 and 2% read all newspapers. What is the probability that a normally selected person ?

(1) Read at least one paper ;

(2) Read exactly one paper ; and

(3) Read no newspaper.

30. A random variable has the following probability function :

X	0	1	2	3	4	5	6	7
P(X) :	0	k	$2k$	$2k$	$3k$	k	$2k$	$7k$

(i) Evaluate P (A) if A = {1 < X < 4}

(ii) Evaluate $P(X < 6)$ and $P(X \geq 6)$.

(iii) If $P(X < a)$ $\frac{1}{2}$, find a.

31. If X and Y are two independent normal variates with means μ_1 and μ_2 and variance σ_1^2 and σ_2^2 respectively. Show that V = X – Y follows normal distribution. Find its mean and variance.

32. (i) Find the expectation of the number on a die when thrown.

(ii) Two dices are thrown ; find the expected values of the sum of number of points on them.

(2 × 10 = 20 marks)

D 11878-A (Pages : 4) Name..

Reg. No......................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Statistics

STS 3C 03—STATISTICAL INFERENCE

(2014–2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

STS 3C 03—STATISTICAL INFERENCE

(Multiple Choice Questions for SDE Candidates)

1. The number of possible samples of size n out of N population units with replacement is :

(A) N^2.
(B) n^2.
(C) ∞
(D) N !

2. If the samples values are 1, 3, 5, 6, 9, the SE of the sample mean is :

(A) SE = $\sqrt{2}$.
(B) SE = $1/\sqrt{2}$.
(C) SE = 2.0.
(D) SE = 1/2.

3. The degrees of freedom for student's 't' based on a random sample of size n is :

(A) $n - 1$.
(B) n.
(C) $n - 2$.
(D) $(n - 1)/2$.

4. Chi-square distribution is used to test :

(A) Goodness of fit.
(B) Hypothetical value of population variance.
(C) Both (A) and (B).
(D) Neither (A) nor (B).

5. F distribution was invented by :

(A) R.A. Fisher.
(B) G.W. Snedecor.
(C) W.Z. Gosset.
(D) J. Neymann.

6. The estimator $\bar{x}$ of population mean is :

(A) an unbiased estimator.
(B) a consistent estimator.
(C) Both (A) and (B).
(D) Neither (A) nor (B).

7. Factorisation theorem for sufficiency is known as :

(A) Rao - Blackwell theorem.
(B) Cramer Rao theorem.
(C) Chapman Robins theorem.
(D) Fisher - Neyman theorem.

8. Least square theory was propounded by whom and in which year ?

(A) Gauss in 1809. (B) Markov in 1900.

(C) Fisher in 1920. (D) None of these.

9. For an estimator to be consistent, the unbiasedness of the estimator is :

(A) Necessary. (B) Sufficient.

(C) Necessary as well as sufficient. (D) Neither necessary nor sufficient.

10. The 100 $(1-\alpha)$% confidence interval for μ of $N(\mu,\sigma)$ when σ unknown, using a sample of size less than 30 is :

(A) $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n-1}}$. (B) $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$.

(C) $\bar{x} \pm t_{\alpha}\frac{s}{\sqrt{n-1}}$. (D) $\bar{x} \pm t_{\alpha}\frac{s}{\sqrt{n}}$.

11. A random sample of 16 housewives has an average body weight of 52 kg and a standard deviation of 3.6 kg. 99% confidence limits for body weight in general are :

(A) (54.66, 49.345). (B) (52.66, 51.34).

(C) (55.28, 48.72). (D) None of the above.

12. Formula for the confidence interval for the ratio of variances of the two normal population involves :

(A) χ^2-distribution. (B) F distribution.

(C) t-distribution. (D) None of the above.

13. An 'hypothesis' means :

(A) Assumption. (B) A testable preposition.

(C) Theory. (D) Supposition.

14. Neymann Pearson lemma provides :

(A) An unbiased test. (B) A most powerful test.

(C) An admissible test. (D) Minimax test.

15. Every test statistic is :

(A) An estimate.
(B) A random variable.
(C) A fixed value.
(D) None of these.

16. Standard error of the difference of proportions $p_1' - p_2'$ in two classes under the hypothesis $H_0 : p_1 = p_2$ with usual notations is :

(A) $\sqrt{p^* q^* \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$.
(B) $\sqrt{p^* \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$.
(C) $p^* q^* \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$.
(D) $\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$.

17. Equality of two population variances can be tested by :

(A) χ^2-test.
(B) t-test.
(C) F-test.
(D) Z-test.

18. The hypothesis that population variance has a specified value can be tested by :

(A) F-test.
(B) Z-test.
(C) χ^2-test.
(D) t-test.

19. When d.f. for χ^2 are 100 or more, χ^2 is approximated to :

(A) t-distribution.
(B) F-distribution.
(C) Z-distribution.
(D) None of these.

20. If the calculated value of χ^2 is greater than its degrees of freedom, then :

(A) Null hypothesis be accepted directly.
(B) Null hypothesis be rejected straight-away.
(C) χ^2 table be consulted to arrive at a decision about the null hypothesis.
(D) All the above.

D 11878 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Statistics

STS 3C 03—STATISTICAL INFERENCE

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in* **one word**.
Each question carries 1 *mark.*

Name the following :

1. An estimator which contains all information about the parameter contained in the sample.
2. The error of accepting null hypothesis when it is false.
3. This distribution is used in testing of independence of attributes.

Fill up the blanks :

4. An efficient estimator is an estimator with minimum ————.
5. ———— is the method of estimating a particular value for an unknown parameter.
6. If X_1 and X_2 are two independent standard normal variables, then $t = \frac{X_1}{X_2}$ follows ————.
7. The statistic used to test the mean of a normal population follows ———— distribution.

Write True *or* False :

8. If $E(t) \neq \theta$, and $V(t)$ tends to infinity as n tends to infinity, then t is a consistent estimator of θ.
9. There may exist more than one unbiased estimators for a parameter.
10. A statistical hypothesis which completely specifies the population is simple hypothesis.

($10 \times 1 = 10$ marks)

Turn over

Section B

Answer **all** *questions in* **one sentence** *each.*
Each one carries 2 marks.

11. Define interval estimation.
12. Define unbiased estimator.
13. Identify the distribution of the ratio of the squares of two independent standard normal random variables.
14. Define power of a test.
15. Define Parameter.
16. State Fisher-Neymaan factorization theorem.
17. What is the test statistics used in small sample test to test the mean of a normal population when σ^2 is unknown ?

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each one carries 4 marks.

18. Obtain the mode of a Chi-square random variable with n degrees of freedom.
19. Distinguish between one tailed and two tailed test.
20. For a Poisson distribution with parameter λ, show that sample mean $\bar{x}$ is the sufficient estimator of λ.
21. Explain the method of moments in estimation.
22. A sample of size 17 taken from $N(\mu, \sigma^2)$. Mean of the sample is 22 and the sample variance is 16. Using the data, find a 90% confidence interval for μ.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each one carries 6 marks.

23. For a random variable of size 16 from $N(\mu, \sigma)$ population, the sample variance is 16. Find a and b such that $P(a < \sigma^2 < b) = 0.60$.

24. Find the mean of a random variable follow t-distribution with n degrees of freedom.

25. Explain the method of M L E. List the properties of a M L Estimator.

26. Derive the confidence interval for the mean of a normal population when population variance is known.

27. State and prove sufficient conditions for a consistent estimator.

28. Explain the method of Chi-square test of independence of attributes.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each one carries 10 *marks.*

29. (i) If F follows F-distribution with (m, n) degrees of freedom. Derive the probability distribution of Y = 1/F.

(ii) Derive any one statistic following F-distribution.

30. Use Neymaan-Pearson Theorem to find a most powerful test with significance level α for testing the hypothesis $H_0 : \mu = \mu_0$ against, $H_1 : \mu = \mu_1, (\mu_1 > \mu_0)$ using a random sample $x_1, x_2, \ldots, x_n$ drawn from the population with pdf $f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{1}{18}(x-\mu)^2}, -\infty < x < \infty$.

31. Explain Chi-square test of goodness of fit. The theory predicts the proportion of four groups A, B, C and D of individuals watching a particular TV programme is 10 : 4 : 4 : 2. In a survey among 1600 individuals, the members in the four groups were 850, 350, 250 and 100. Does the data support the ratio suggested ?

32. (i) Explain the paired t-test of equality of means of two normal populations when the population standard deviations are unknown and the sample size is small.

(ii) Marks obtained by two sets of 8 students undergone two different type of training mode is given below :

Diet A	23	30	40	35	26	36	25	28
Diet B	28	35	32	38	25	31	30	29

Test whether the trainings are different as far as the marks after the trainings are concerned, ts at 5% level of significance.

(2 × 10 = 20 marks)

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THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Statistics

STS 3B 03—STATISTICAL ESTIMATION

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions in one word.*
Each question carries 1 mark.

Name the following :

1. Functions representing sample characteristics.
2. A function of the sample observations which contains as much informations about the unknown parameter contained in the sample.
3. X_1 and X_2 are two independent chi-square variate having n_1 and n_2 degrees of freedom. Then $\frac{X_1 / n_1}{X_2 / n_2}$ follows.

Fill up the blanks :

4. The Variance of a chi - square distribution with mean 4 is ————.
5. Interval for the value of an unknown parameter with a specified probability is called ————.
6. Inequality which gives the lower limit for the variance of an unbiased estimator is ————.
7. The ratio of two independent standard normal random variables is ————.

Write True *or* False :

8. Fisher-Neymann theorem helps to obtain sufficient estimator.
9. The maximum likelihood estimator of λ in poisson distribution is sample median.
10. The distribution of the reciprocal of X following $F(m, n)$ is $F(n, m)$.

(10 × 1 = 10 marks)

Turn over

Section B

Answer **all** *questions in one sentence each.*
Each question carries 2 marks.

11. Define chi-square distribution with n degrees of freedom.
12. What is meant by a sufficient estimator ?
13. Give any two uses of F-distribution.
14. State Rao-Blackwell theorem.
15. Define efficient estimator.
16. State properties of maximum likelihood estimator.
17. Give confidence interval for proportion P.

(7 × 2 = 14 marks)

Section C

Answer any **three** *questions.*
Each question carries 4 marks.

18. Distinguish between point estimation and interval estimation.
19. Derive mode of chi-square distribution.
20. Explain the method of minimum variance.
21. State and prove sufficient condition for consistency.
22. Briefly explain Bayesian estimation.

(3 × 4 = 12 marks)

Section D

Answer any **four** *questions.*
Each question carries 6 marks.

23. Define unbiasedness. Show that, if $X_1, X_2 \ldots\ldots X_n$ is a random sample from $B(1, p)$ population, the estimator $\frac{\Sigma X_i (\Sigma X_i - 1)}{n(n-1)}$ unbiased for p^2.

24. Explain the method of maximum likelihood estimator. If $f(x) = \frac{1}{b-a}$; $a \le x \le b$. Find the MLE's of 'a' and 'b'.

25. Derive mgf of chisquare distribution. Hence show that chi square distribution possesses additive property.

26. State and prove tchebychev's inequality.

27. Derive $(1-\alpha)$ 100% confidence interval for difference of population means assuming small sample and identical variance.

28. Define method of moments. Obtain the moment estimator of θ in $f(x)=\frac{1}{\theta}e^{-x/\theta}, x>0$.

(4 × 6 = 24 marks)

Section E

Answer any **two** *questions.*
Each question carries 10 *marks.*

29. Define t, F and chisquare distributions and establish the inter relation between them.

30. Define the efficiency of the estimate. Let X_1, X_2 and X_3 be a sample from a population with mean μ and variance σ^2. Consider the statistic $t_1=\frac{X_1+X_2+X_3}{3}$ and $t_2=\frac{2X_1-X_2+X_3}{2}$. Compare the efficiency of the estimators.

31. Derive MLE of :

    (i) μ when σ^2 is known and

    (ii) σ^2 when μ is known, if a random sample is taken from a normal population $N(\mu,\sigma^2)$.

32. Derive $(1-\alpha)$ 100% confidence interval for μ when a random sample is taken from a normal population $N(\mu,\sigma^2)$. when,

    (i) σ^2 is known.

    (ii) σ^2 is unknown.

(2 × 10 = 20 marks)

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 01—OPERATION RESEARCH – I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Weighage 2 for each question.

1. What is linear programming problem ? Explain its basic characteristics.
2. Explain sequencing problem. What are the elements that characterize a sequencing problem ?
3. Define transportation problem. Give its general mathematical model.
4. Define (i) basic solution ; and (ii) degenerate solution. Show that the system of equations $2x + y - z = 2$ and $3x + 2y + z = 3$ has a degenerate solution.
5. What is duality in LPP ? State the functional properties of duality in LPP.
6. Define (i) two person zero sum game ; (ii) payoff matrix ; (iii) pure and mixed strategies ; and (v) optimum strategy.
7. Define different types of integer programming problems. Mention its applications in practical situations.
8. Explain the role of sensitivity analysis in linear programming. Under what circumstances it is needed ?

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Weightage 3 for each question.

9. Explain what is meant by degeneracy and cycling in linear programming problems. How is this effect removed ?

10. Find the optimal sequence where the processing times in hours required to complete the tasks on two machines are given below :

Task	A	B	C	D	E	F
Machine A	4	8	3	6	7	5
Machine B	6	33	7	2	8	4

11. Describe the graphical method of solving $2 \times n$ and $m \times 2$ games.

12. Define (i) slack ; (ii) surplus ; and (iii) artificial variables. Explain two phase simplex method of solving LPP.

13. Describe the dual simplex method of solving LPP. Give the situation where dual simplex method will be applicable?

14. In a textile sales emporium, four salesmen A, B, C and D are available to four counters W, X, Y and Z. Each salesman must handle only one counter. The service (in hours) of each counter when manned by each salesman is given below :

		Salesmen			
		A	B	C	D
Counters	W	41	72	39	52
	X	22	29	49	65
	Y	27	39	60	51
	Z	45	50	48	52

How should the salesmen be assigned to counters so as to minimize the service time ?

$(4 \times 3 = 12$ weightage)

Part C

Answer any **two** *questions.*
Weightage 5 *for each question.*

15. (i) Write short notes on simplex method in linear programming problem.

    (ii) Show that the set of all feasible solutions (if exist) of a linear programming problem forms a convex set.

    (iii) Prove that if an LPP has a feasible solution, then it also has a basic feasible solution.

16. Solve the following all integer programming problem using Gomory's cutting plane algorithm :

    Maximize $Z = 7x_1 + 9x_2$

    Subject to the constraints $3x_2 - x_1 \le 6$, $7x_1 + x_2 \le 35$ and $x_1, x_2 \ge 0$ and integers.

17. Explain the principles of dominance to solve the game. Using dominance property, solve the game whose payoff matrix is given below :

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	3	2	4	0
	A_2	3	4	2	4
	A_3	4	2	4	0
	A_4	0	4	0	8

18. Write short notes on the following :

    (i) Vogels Approximation Method.

    (ii) Zero-One programming problem.

    (iii) Big M method.

    (iv) Minimax and Maximin Principle.

(2 × 5 = 10 weightage)