Ph.D THESIS MATHEMATICS

ON PROPERTIES OF REGULAR OPEN SETS AND COMPARISON BETWEEN FUNCTIONS

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By

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CERTIFICATE

This is to certify that the thesis entitled "**ON PROPERTIES OF REGULAR OPEN SETS AND COMPARISON BETWEEN FUNCTIONS**" submitted by **Anuradha N** to St.Joseph's College (Autonomous), Devagiri for the award of the degree of **Doctor of Philosophy** is a bona-fide record of the research work carried out by her under my supervision and guidance. The contents of this thesis, in full or parts, have not been submitted and will not be submitted to any other Institute or University for the award of any degree or diploma.

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DECLARATION

This thesis entitled "**ON PROPERTIES OF REGULAR OPEN SETS AND COMPARISON BETWEEN FUNCTIONS**" contains no material which has been in full or parts submitted and will be submitted to any other Institute or University for the award of any degree or diploma. To the best of my knowledge and belief, it contains no material previously published by any other person except where due reference is made in text of thesis.

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CONTENTS

ANURADHA N. “ON PROPERTIES OF REGULAR OPEN SETS AND COMPARISON BETWEEN FUNCTIONS.” THESIS. CENTRE FOR RESEARCH & PG STUDIES IN MATHEMATICS, ST. JOSEPH’S OLLEGE(AUTONOMOUS), DEVAGIRI, UNIVERSITY OF CALICUT, 2018.

INTRODUCTION

Concept of the regular open set was introduced by M. H. Stone in 1937. It has been shown by him that the collection of all regular open sets in a topological space form a complete lattice. M. H. Stone further gave results on application of theory of Boolean algebras to general topology. In 1980, R. C. Jain worked on 'role of regularly open sets in topology' on his thesis. C. Ronse in 1990 studied generalizations of regular open sets in a complete lattice and discussed on relevance of such concepts for representing objects in continuous and digital spaces. Thus introduction of regular open sets raised many topological questions which have led to a productive study in which many new notions have been defined and examined. As a result of which many new properties and characterizations have been introduced. Purpose of this thesis is to investigate more on regular open sets, its types, properties, characterizations and also to compare various functions.

In the years 2001 and 2003, F. Nakaoka and N. Oda [20,21,22] introduced and studied minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets. S. S. Benchali, Basavaraj Ittanagi and R. S. Wali [5] studied on minimal open sets and functions in topological spaces. Whether there exists such minimal and maximal sets in collection of regular open sets is enquired in first chapter. Also discussion on minimal regular open, maximal regular open, minimal regular closed and maximal regular closed sets are given. In many examples, sets which have only X as a regular open superset are seen. Based on that property weakly regular open sets are defined. In 1983, A. S. Mashhour[18] introduced the concept of supra topological space[18] and studied $s-continuous$ maps and $s^*-continuous$ maps. In 2008, R. Devi [9] introduced and studied a class of sets called supra α-open sets [9] and class of maps on topological spaces called supra α-continuous maps. Attempt is made to find out whether such sets and functions can be defined using regular open sets. Supra r-open sets are defined and properties are discussed. Thus four new types of regular open sets namely minimal regular open, maximal regular open, weakly regular open and supra r-open sets are introduced in chaper 1.

Two distinct points and two distinct sets of a topological space can be separated using open sets and closed sets. Separation axioms were defined for that purpose. Questions like whether such separation is possible using regular open sets and regular closed sets led to the introduction of separation axioms using such sets. Using those axioms, points and sets are separated in terms of regular open, regular closed and clopen sets. While coming across types of regular open sets, some spaces are seen which contain only minimal regular open sets or maximal regular open sets or no proper regular open sets. To categorize such spaces, rT_{min}, rT_{max} and rT_{weak} spaces are introduced. Thus using types of regular open sets, special types of spaces are introduced in chapter 2.

Studies on various types of regular open sets and special spaces posed problems like whether there exist certain special functions defined on such spaces. Such a function named - almost perfectly continuous was studied by Dontchev, Ganster and Reilly. Some properties of almost perfectly continuous function was studied by D. Singh[28]. In chapter 3, some attempt is done to study more properties of almost perfectly continuous functions. Another type of function namely somewhat continuous was studied by Gentre and Hoyle [11]. Using regular open sets, somewhat r-continuous function is thus defined. S. S. Benchali, Basavaraj Ittanagi and R. S. Wali [5] studied on minimal continuous maps. Studies using regular open sets in this area resulted in many functions which maps various types of regular open sets and open sets in topological spaces. Using the notion of supra continuity, supra r-continuity is defined. Thus many functions which map various types of regular open sets to themselves and each other are introduced in this chapter. After studying such functions, it has been found that certain functions imply certain other functions. So attempt is made to find out whether there exists any such relation between above defined functions and some other existing functions. What will happen to the composition and restriction of these functions are also studied in chapter 3.

Relation between continuity, openness, closedness and invertibility of functions are given in many books on topology. In chapter 4, whether such relation exists between various functions defined in chapter 3 is checked. Relation between various such functions in certain special spaces is also studied. Relation between various functions and their graph functions has been studied by several researchers. So attempt is made to find out the relation between the various functions defined above and their graph functions.

Whole work is divided into 4 chapters. Chapters are divided into sections and sub sections. In chapter 1, we introduce certain new types of regular open sets. The chapter contains 9 sections. Section 2, contains preliminary ideas on regular open sets. Minimal regular open sets and maximal regular open sets are introduced in section 3. In section 4, minimal regular closed sets and maximal regular closed sets are introduced. Section 5 and 6 deals with properties of sets discussed in sections 3 and 4. In section 7, weakly regular open sets are introduced and some of its properties are studied. Section 8 contains preliminary ideas on supra topology. In section 9, discussion is done on supra r-open sets.

In chapter 2, we define separation axioms using regular open sets. Spaces like rT_{min}, rT_{max} and rT_{weak} are introduced and their properties are studied. Also relation between the spaces rT_{min}, rT_{max} and rT_{weak} and some other spaces like r-door, $rT_{\frac{1}{2}}$ etc. are studied. Section 2, contains preliminary ideas on separation axioms. In section 3, more separation axioms in terms of regular open sets are given. Section 4, introduces quasi regular components. Submaximal regular spaces, r-door and $rT_{\frac{1}{2}}$ are introduced in section 5. Section 6, contains discussion on regular open and regular closed functions. Properties of rT_2, r-regular and r-normal spaces are studied in section 7. Relation between various spaces is the topic of section 8. In section 9, rT_{min}, rT_{max} and rT_{weak} spaces are introduced and properties are studied.

Chapter 3 contain discussions about almost perfectly continuous functions, somewhat r-continuous functions, minimal and maximal r-continuous functions minimal-maximal and maximal-minimal r-continuous functions, minimal and maximal r-irresolute functions. Properties of composition, restriction and extension of such functions is also studied. Throughout the chapter X and Y denote topological spaces with topologies τ and σ respectively. Section 2 is on preliminary ideas. In section 3, properties of almost perfectly continuous function is discussed. Somewhat r-continuous function and its properties are given in section 4. Section 5, is on minimal r-continuous and maximal r-continuous functions and their properties. Supra r-continuous function and its properties is discussed in section 6.

In chapter 4, properties of almost perfectly continuous function and somewhat r-continuous function on certain special spaces like r-door, $rT_{\frac{1}{2}}$ etc. are discussed. Discussion is also done on regular totally open function, some what r-open function, supra r-open function, supra r-closed function and minimal r-open function. Properties of almost perfectly continuous function and somewhat r-continuous function, on certain special spaces are studied in section 2. Regular totally open function on special spaces is discussed in section 3. Section 4, is on somewhat r-open function. Supra r-open function is the topic of discussion of section 5. Minimal r-open function and its properties are introduced in section 6. Properties of graph function of various functions are given in section 7.

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CHAPTER 1

Types of regular open sets

1.1 Introduction

In this chapter, we introduce certain new types of regular open sets. The chapter contains 9 sections. Section 2 contains preliminary ideas on regular open sets. Minimal regular open sets and maximal regular open sets are introduced in section 3. In section 4, minimal regular closed sets and maximal regular closed sets are introduced. Section 5 and 6 deals with properties of sets discussed in sections 3 and 4. In section 7, weakly regular open sets are introduced and some of its properties are studied. Section 8 contains preliminary ideas on supra topology. In section 9, discussion is done on supra r-open sets.

1.2 Preliminary ideas on regular open sets

Definition 1.2.1

A subset A of a topological space X is said to be

(i.) *regular open, if $A = Int(Cl(A))$.*

(ii.) *regular closed, if $A = Cl(Int(A))$.*

(iii.) *clopen, if A is both open and closed.*

1.2.1 Properties of regular open sets

(i.) Every clopen set is regular open and every regular open set is open.

(ii.) Finite union of regular open sets need not be regular open.

(iii.) Finite intersection of regular open sets is regular open.

(iv.) Arbitrary union of regular open sets need not be regular open.

(v.) Arbitrary intersection of regular open sets is regular open.

1.3 Minimal regular open and maximal regular open sets

Definition 1.3.1

A proper non empty regular open subset U of a topological space X is said to be a minimal regular open set, if any regular open set which is contained in U is ϕ or U.

Example 1.3.1

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a, c\}\}$. Then $\{a\}, \{b\}$ and $\{c\}$ are minimal regular open sets.

Definition 1.3.2

A proper non empty regular open subset U of a topological space X is said to be a maximal regular open set, if any regular open set which contains U is U or X.

Example 1.3.2

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a, c\}\}$. Then $\{b, c\}, \{a, c\}$ and $\{a, b\}$ are maximal regular open sets.

1.4 Minimal regular closed and maximal regular closed sets

Definition 1.4.1

A proper non empty regular closed subset F of a topological space X is said to be a minimal regular closed set, if any regular closed set which is contained in F is ϕ or F.

Example 1.4.1

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a, c\}\}$. Then $\{a\}, \{b\}$ and $\{c\}$ are minimal regular closed sets.

Definition 1.4.2

A proper non empty regular closed subset F of a topological space X is said to be a maximal regular closed set, if any regular closed set which contains F is F or X.

Example 1.4.2

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b\}, \{c\}, \{b\}, \{a, c\}\}$. Then $\{b, c\}, \{a, c\}$ and $\{a, b\}$ are maximal regular closed sets.

1.5 Properties of minimal regular open and maximal regular open sets

Theorem 1.5.1

Let X be a topological space and $U \subset X$. Then U is a minimal regular open set if and only if $X - U$ is a maximal regular closed set and U is a maximal regular open set if and only if $X - U$ is a minimal regular closed set.

Theorem 1.5.2

Let U be a minimal regular open set and W be a regular open set.

Then either $U \cap W = \phi$ or $U \subset W$.

Theorem 1.5.3

Let U and W be minimal regular open sets. Then either $U \cap W = \phi$ or $U = W$.

Theorem 1.5.4

Let U be a maximal regular open set and W be a regular open set.

Then either $U \cup W = X$ or $W \subset U$.

Theorem 1.5.5

Let U and W be maximal regular open sets.

Then either $U \cup W = X$ or $U = W$

1.6 Properties of minimal regular closed and maximal regular closed sets

Theorem 1.6.1

Let X be a topological space and $F \subset X$. Then F is a minimal regular closed set if and only if $X - F$ is a maximal regular open set and F is a maximal regular closed set if and only if $X - F$ is a minimal regular open set.

Theorem 1.6.2

Let U be a minimal regular closed set and W be a regular closed set. Then either $U \cap W = \phi$ or $U \subset W$.

Theorem 1.6.3

Let U and W be minimal regular closed sets. Then either $U \cap W = \phi$ or $U = W$

Theorem 1.6.4

Let U be a maximal regular closed set and W be a regular closed set.

Then either $U \cup W = X$ or $U \supset W$.

Theorem 1.6.5

Let U and W be maximal regular closed sets. Then either $U \cup W = X$ or $U = W$.

1.7 Weakly regular open sets

Definition 1.7.1

Let A be a proper subset of X. Then A is said to be weakly regular open, if the only regular open set containing A is X.

Example 1.7.1

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a, b\}$ is a weakly regular open set.

Definition 1.7.2

Let A be a proper subset of X. Then A is said to be weakly regular closed, if its complement is weakly regular open.

1.7.1 Properties of weakly regular open and weakly regular closed sets

Remark 1.7.0:

(i.) Union of two proper regular open sets is either weakly regular open or whole set.

(ii.) Intersection of two proper regular closed sets is either weakly regular closed or empty.

(iii.) Intersection of a weakly regular open set and a proper regular open set is regular open.

(iv.) Union of a weakly regular closed set and a proper regular closed set is regular closed.

Remark 1.7.0:

(i.) Union of two weakly regular open sets is either a weakly regular open set or the whole set.

(ii.) Intersection of two weakly regular closed sets is either a weakly regular closed set or empty.

(iii.) Union of two weakly regular closed sets is either a closed set or the whole set.

(iv.) Intersection of two weakly regular open sets is either an open set or empty.

1.8 Preliminary ideas on supra topology

Definition 1.8.1

Let X be any set. A collection $\tau*$ of subsets of X is called a supra topology [18] on X, if

$X, \phi \in \tau*$ and $\tau*$ is closed under arbitrary union. $(X, \tau*)$ is called a supra topological space. The elements of $\tau*$ are known as supra open sets. The complement of a supra open set is known as supra closed set.

Definition 1.8.2

Supra $Int(A)$ is the union of all supra open sets contained in A. Supra $Cl(A)$ is the intersection of all supra closed sets containing A.

Remark 1.8.0:

If (X, τ) is a topological space and $\tau \subset \tau*$, then $\tau*$ is known as supra topology associated with τ.

1.9 Supra r-open sets

Definition 1.9.1

Let (X, τ^*) be a supra topological space. A is called Supra r-open if $A =$ Supra $Int(Cl(A))$, where Supra $Int(Cl(A))$ denotes Int(Cl(A)) in $\tau*$. The complement of a supra r-open set is called a supra r-closed set.

Example 1.9.1

Let $(X, \tau*)$ where $X = \{a, b, c, d\}$, $\tau* = \{X, \phi, \{a\}, \{b, c, d\}, \{a, b\}\}$ be a supra topological space. Then $\{b, c, d\}$ is supra r-open.

Remark 1.9.0:

Let (X, τ) be a topological space and τ^* be supra topology associated with τ. Then every regular open set is supra r-open.

Theorem 1.9.1

Every supra r-open set is supra open.

Proof:

Since every regular open set is open, supra r-open set is supra open. □

Remark 1.9.1:

Converse of the above theorem need not be true.

Example 1.9.2

Let $(X, \tau*)$ *where* $X = \{a, b, c, d\}$, $\tau* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ *be a supra topological space. Then* $\{a, b\}$ *is a supra open set, but not supra r-open.*

Theorem 1.9.2

If supra topology equals discrete topology, then every supra open set is supra r-open.

Remark 1.9.2:

(i.) Union of a Supra r-open set and a supra open set is a supra open set as supra topology is closed under arbitrary unions.

(ii.) Intersection of a Supra r-open set and a supra open set need not be a supra open set as supra topology is not closed under intersection.

Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}, \tau* = \{X, \phi, \{a\}, \{b, c\}, \{a, c\}\}$.

Then $\{b, c\}$ is supra r-open. $\{a, c\}$ is supra open.

But their intersection $\{c\}$ is not supra open.

Theorem 1.9.3

Finite intersection of supra r-open sets is supra r-open.

Proof:

Let V_1 and V_2 be supra r-open. Then $V_1 = \text{Supra Int } (\text{Cl } (V_1)), V_2 = \text{Supra Int } (\text{Cl } (V_2))$.

$\text{Supra Int } (\text{Cl } (V_1 \cap V_2)) \subseteq Supra\ Int(Cl(V_1)) \cap Supra\ Int(Cl(V_2)) {=} V_1 \cap V_2$.

Also $V_1 \cap V_2 \subseteq \text{Supra Int } (\text{Cl}(V_1 \cap V_2))$. Hence $V_1 \cap V_2$ is supra r-open. □

Theorem 1.9.4

Finite union of supra r-closed sets is supra r-closed.

Proof:

Let V_1 and V_2 be supra r-closed. Then $(X - V_1) \cap (X - V_2)$ is supra r-open. That is $X - (V_1 \cup V_2)$ is supra r-open. Hence $V_1 \cup V_2$ is supra r-closed. □

Theorem 1.9.5

Finite union of supra r-open sets may fail to be supra r-open.

Example 1.9.3

Let $X = \{a, b, c\},\ \tau * = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Then $\{a\}$ *and* $\{b\}$ *are supra r-open. But their union* $\{a, b\}$ *is not supra r-open.*

Theorem 1.9.6

Finite intersection of supra r-closed sets may fail to be supra r-closed.

Example 1.9.4

Let $X = \{a, b, c\}$, $\tau * = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. *Then* $\{b, c\}$ *and* $\{a, c\}$ *are supra r-closed. But* *their intersection* $\{c\}$ *is not supra r-closed.*

1.9.1 Supra r-closure and supra r-interior

Definition 1.9.2

Supra r-closure of a set A denoted by Supra rCl (A) is the intersection of all supra r-closed sets containing A.

Example 1.9.5

Let $X = \{a, b, c\}, \ \tau* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. *Then Supra rCl (*$\{a\}$*)=* $\{a,c\}$.

Definition 1.9.3

Supra r-interior of a set A denoted by Supra rInt (A) is the union of all supra r-open sets contained in A.

Example 1.9.6

Let $X = \{a, b, c\}, \ \tau* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Then Supra rInt ($\{a\}$*)=* $\{a\}$.

Remark 1.9.6:

(i) Supra rInt (A) is a supra r-open set.

(ii) Supra rCl (A) is a supra r-closed set.

Theorem 1.9.7

(i) Supra $rInt(A) \subseteq A$ *and equality holds if and only if A is a supra r-open set.*

(ii) $A \subseteq Supra\ \ rCl(A)$ *and equality holds if and only if* A *is a supra r-closed set.*

Theorem 1.9.8

(i) $X - Supra\ \ rInt(A) = Supra\ \ rCl\,(X - A)$.

(ii) $X - Supra\ \ rCl(A) = Supra\ \ rInt\,(X - A)$.

Theorem 1.9.9

(i) $Supra\ \ rInt(A \cap B) = Supra\ \ rInt\,(A) \cap Supra\ \ rInt\,(B)$.

(ii) $Supra\ \ rCl\,(A \cup B) = Supra\ \ rCl(A) \cup Supra\ \ rCl(B)$.

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CHAPTER 2

Separation axioms in terms of regular open sets

2.1 Introduction

In this chapter, we define separation axioms using regular open sets. Spaces like rT_{min}, rT_{max} and rT_{weak} are introduced and their properties are studied. Also relation between the spaces rT_{min}, rT_{max} and rT_{weak} and some other spaces like r-door, $rT_{\frac{1}{2}}$ etc. are studied. Section 2 contains preliminary ideas on separation axioms. In section 3, more separation axioms in terms of regular open sets are given. Section 4, introduces quasi regular components. Submaximal regular, r-door and $rT_{\frac{1}{2}}$ spaces are introduced in section 5. Section 6 contain discussion on regular open and regular closed functions. Properties of rT_2, r-regular and r-normal spaces are studied in section 7. Relation between various spaces is the topic of section 8. In section 9, rT_{min}, rT_{max} and rT_{weak} spaces are introduced and their properties are studied.

2.2 Preliminary ideas on separation axioms

Definition 2.2.1

A topological space X is said to be

(i.) δT_0 [13], if for each pair of distinct points x and y in X, there exists a regular open

set which contains one of the points x and y, but not the other.

(ii.) δT_1 *(respectively clopen T_1)([15],[10]), if for each pair of distinct points x and y in X, there exists regular open sets (respectively clopen sets) U and V containing x and y respectively such that $x \in U, y \notin U$ and $y \in V, x \notin V$.*

2.3 More about separation axioms in terms of regular open sets.

Definition 2.3.1

A topological space X is said to be

(i.) *rT_2, if every two distinct points of X can be separated by disjoint regular open sets.*

(ii.) *r-regular, if for each closed set F of X and a point $x \notin F$, there exist disjoint regular open sets U and V such that $F \subset U$ and $x \in V$.*

(iii.) *r-normal, if each pair of non empty disjoint closed sets can be separated by disjoint regular open sets.*

(iv.) *ultra Hausdorff, if every two distinct points of X can be separated by disjoint clopen sets.*

(v.) *ultra regular, if for each closed set F of X and a point $x \notin F$, there exist disjoint clopen sets U and V such that $F \subset U$ and $x \in V$.*

(vi.) *ultra normal, if each pair of non empty disjoint closed sets can be separated by disjoint clopen sets.*

(vii.) *ro-regular, if for each regular closed set F of X and a point $x \notin F$, there exist* *disjoint regular open sets U and V such that $F \subset U$ and $x \in V$.*

(viii.) ro-normal, if for each pair of disjoint regular closed sets U and V of X, there exist disjoint regular open sets G and H such that $U \subset G$ and $V \subset H$.

(ix.) clopen regular, if for each clopen set F of X and a point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

(x.) clopen normal, if for each pair of disjoint clopen sets U and V of X, there exist disjoint open sets G and H such that $U \subset G$ and $V \subset H$.

2.4 Quasi regular components

Definition 2.4.1

Let X be a topological space and $x \in X$. Then the set of all points y in X such that $x \in U, y \in V$ and $U \cap V \neq \phi$, where U and V are regular open sets or regular closed sets of X is said to be quasi regular component of x.

Theorem 2.4.1

If a space has quasi regular components, then it cannot be rT_2.

Proof:

Proof follows from the definition of quasi regular component and rT_2. □

2.5 r-door, $rT_{\frac{1}{2}}$ and *submaximal regular space*.

Definition 2.5.1

A topological space (X, τ) is called an r−door space, if every subset is either regular closed or regular open in (X, τ).

Example 2.5.1

Let $X = \{a, b, c\}$, $\tau = \{X,\ \phi,\ \{a\},\ \{b,\ c\}\}$. Then X is an $r -$ door space.

Definition 2.5.2

$A \subset X$ is called $r -$ dense if $rCl(A) = X$.

Definition 2.5.3

A topological space (X, τ) is called a submaximal regular space if every $r -$dense subset of (X, τ) is regular open.

Example 2.5.2

Let $X = \{a, b, c\}$, $\tau = P(X)$. Then X is a submaximal regular space

Definition 2.5.4

A topological space (X, τ) is called an $rT_{\frac{1}{2}}$ space, if every closed subset of (X, τ) is regular closed in (X, τ) .

Example 2.5.3

Let $X = \{a, b, c\}, \tau = \{X,\ \phi,\ \{a\},\ \{b,\ c\}\}$. Then X is an $rT_{\frac{1}{2}}$ space.

2.6 Regular open and regular closed functions.

Definition 2.6.1

A function $f : X \to Y$ is

(i.) regular open, if $f(U)$ is a regular open set in Y, for every open set U in X.

(ii.) regular closed, if $f(F)$ is a regular closed set in Y, for every closed set F in X.

2.6.1 Properties of regular open and regular closed functions

Theorem 2.6.1

Every regular open function is an open function and every regular closed function is a closed function.

Remark 2.6.1:

Converse of the above theorem need not be true.

Example 2.6.1

Let $X = Y = \{a, b, c\}, \tau = \sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be the identity function. Then f is an open and closed function. But f is not a regular open and regular closed function.

Theorem 2.6.2

If X is T_2 and $f : X \to Y$ is a bijective regular open function, then $f(X)$ is rT_2.

Proof:

Let y_1 and y_2 be distinct points of $f(X)$. Since f is surjective, there exists x_1 and x_2 in

X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is T_2, there exists open sets U and V such that $x_1 \in U,\ x_2 \in V$ and $U \cap V = \phi$. Then $y_1 \in f(U)$ and $y_2 \in f(V)$. Also since f is injective, $f(U) \cap f(V) = f(U \cap V) = \phi$. Since f is regular open, $f(U)$ and $f(V)$ are regular open sets. So $f(X)$ is rT_2. □

2.7 Properties of rT_2, r-regular and r-normal spaces

Theorem 2.7.1

Every rT_2 space is T_2.

Proof:

Proof follows from the result that every regular open set is open. □

Theorem 2.7.2

For a topological space X, the following statements are equivalent.

(i.) X is r-regular.

(ii.) For any $x \in X$ and any open set G containing x, there exists a regular open set U in X such that $x \in U$ and $rcl\, U \subset G$.

Proof:

$(i) \Rightarrow (ii)$

Suppose X is r-regular, $x \in X$ and G be any open set containing x. Then $X - G$ is closed in X and $x \notin X - G$. So by r-regularity, there exists regular open sets U and V containing x and $X - G$ such that $x \in U, X - G \subset V$ and $U \cap V = \phi$. Then $U \subset X - V$ and hence

$rcl\, U \subset X - V \subset G.$

$(ii) \Rightarrow (i)$

Suppose (ii) holds. Let $x \in X$ and C be a closed set not containing $x.$ Then $X - C$ is open in $X.$ So by (ii), there exists a regular open set U containing x such that $rcl\, U \subset X - C$ That is $C \subset V = X - rcl\, U,$ a regular open set. Also $U \cap V = \phi.$ Hence (i) holds. □

Theorem 2.7.3

r-regularity is a hereditary property.

Proof:

Suppose X is an r-regular space and Y is a subspace of $X.$ Let $y \in Y$ and D be a closed subset of Y not containing $y.$ Then D is of the form $D = C \cap Y,$ where C is a closed subset of $X.$ Also $y \notin C.$ Hence by r-regularity of $X,$ there exist regular open sets U and V containing y and C such that $y \in U, C \subset V$ and $U \cap V = \phi.$ Let $G = U \cap Y$ and $H = V \cap Y.$ Then G and H are regular open in Y in the relative topology on $Y.$ Also $y \in G, D \subset H$ and $G \cap H = \phi.$ So Y is also r-regular. □

Theorem 2.7.4

For a topological space X, the following statements are equivalent.

(i.) *X is r-normal.*

(ii.) *For any closed set C and any open set G containing C, there exists a regular open set H such that $C \subset H$ and $rcl\, H \subset G$.*

(iii.) *For any closed set C and any open set G containing C, there exists a regular open set H and a regular closed set K such that $C \subset H \subset K \subset G$.*

Proof:

(i) ⇒ (ii).

Suppose that X is r-normal. Let C and $X - G$ be any two closed sets. Then there exists regular open sets U and V such that $C \subset U$ and $X - G \subset V$ and $U \cap V = \phi$. $X - G \subset V$ implies $X - V \subset G$. But $U \subset X - V \subset G$. This implies $rcl\, U \subset G$.

(ii) ⇒ (iii)

Put $K = rCl\, H$ in (ii). Then $C \subset H \subset K \subset G$.

(iii) ⇒ (i)

Let C and D be two closed sets. Then by (iii), for C and $X - D$, there exists regular open set H and regular closed set K such that $C \subset H \subset K \subset X - D$. That is $C \subset H$ and $X - K \supset D$. Hence X is r-normal. □

Theorem 2.7.5

r-normality is a weakly hereditary property.

Proof:

Let X be an r-normal space and Y be a closed subspace of X. Let C and D be two disjoint closed subsets of Y. Since Y is closed, C and D are closed in X. Since X is r-normal, there exists regular open sets U and V in X such that $C \subset U, D \subset V$ and $U \cap V = \phi$. Also $U \cap Y$ and $V \cap Y$ are regular open in Y and $C \subset U \cap Y$ and $D \subset V \cap Y$. Hence Y is r-normal. □

2.8 Relation between various spaces

Theorem 2.8.1

Every ultra Hausdorff space is rT_2.

Proof:

Result holds since clopen sets are regular open. □

Theorem 2.8.2

Every locally indiscrete rT_2 *space is ultra Hausdorff.*

Proof:

Result holds since regular open sets in a locally indiscrete space are clopen. □

Theorem 2.8.3

Every r-normal space is ro-normal.

Proof:

Result holds since regular closed sets are closed. □

Theorem 2.8.4

Every ro-normal, locally indiscrete space is r-normal.

Proof:

Result holds since closed sets in a locally indiscrete space are clopen and clopen sets are regular closed. □

Theorem 2.8.5

Every r-regular space is ro-regular.

Proof:

Result holds since regular closed sets are closed. □

Theorem 2.8.6

Every ro-regular, locally indiscrete space is r-regular.

Proof:

Result holds since closed sets in a locally indiscrete space are clopen and clopen sets are regular closed.. □

Theorem 2.8.7

Every ultra regular space is r-regular.

Proof:

Result holds since clopen sets are regular open. □

Theorem 2.8.8

Every r- regular, locally indiscrete space is ultra regular.

Proof:

Result holds since regular open sets in a locally indiscrete space are clopen. □

Theorem 2.8.9

Every ultra normal space is r-normal.

Proof:

Result holds since clopen sets are regular open . □

Theorem 2.8.10

Every r-normal, locally indiscrete space is ultra normal.

Proof:

Result holds since regular open sets in a locally indiscrete space are clopen. □

2.9 rT_{min}, rT_{max} and rT_{weak} spaces

Definition 2.9.1

A topological space (X, τ) *is said to be an* rT_{min} *space, if every non empty proper regular open subset of* X *is a minimal regular open set.*

Definition 2.9.2

A topological space (X, τ) *is said to be an* rT_{max} *space, if every non empty proper regular open subset of* X *is a maximal regular open set.*

Definition 2.9.3

A topological space (X, τ) *is said to be an* rT_{weak} *space, if every non empty proper open subset of* X *is a weakly regular open set.*

2.9.1 Properties of rT_{min}, rT_{max} and rT_{weak} spaces

Remark 2.9.0:

(i.) rT_{min} and rT_{max} spaces will contain regular open sets of the form $A, X - A$ along with other open sets.

(ii.) rT_{weak} spaces will be of the form $\{\phi,\ A,\ X\}$.

Theorem 2.9.1

A topological space (X, τ) is an rT_{min}(respectively rT_{max}) space if and only if every non empty proper regular closed subset of X is a maximal regular closed (respectively minimal regular closed) set in X.

Proof:

The proof follows from the definition of rT_{min} (rT_{max}) space and from the fact that complement of every minimal regular open (respectively maximal regular open) set is a maximal regular closed (respectively minimal regular closed) set. □

Theorem 2.9.2

Every distinct minimal regular open (respectively maximal regular open) sets in rT_{min} (respectively rT_{max}) space are disjoint.

Theorem 2.9.3

Union of any two distinct maximal regular open sets in an rT_{max} space is whole set.

Theorem 2.9.4

Intersection of any two distinct minimal regular open sets in an rT_{min} space is empty.

Theorem 2.9.5

Let X be an rT_{min} space and Y be a regular open subspace of X. Then Y is also an rT_{min} space.

Proof:

Let Y be a regular open subspace of an rT_{min} space $X.$ Suppose U is a minimal regular open set in X and not a minimal regular open subset of $Y.$ Then there exists a regular open set $V \neq \phi$ in Y such that $V \subset U \subset Y.$ V is regular open in Y implies that V is regular open in $X,$ a contradiction to the fact that U is a minimal regular open set in $X.$ So U is a minimal regular open set in Y and therfore Y is an rT_{min} space. □

Remark 2.9.5:

rT_{min} (respectively rT_{max}) space need not be δT_0 (respectively $\delta T_1,\ rT_2$) and vice-versa.

Example 2.9.1

(i.) *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. (X, τ) is an rT_{min}(respectively rT_{max}) space; but it is not a δT_0 (respectively $\delta T_1,\ rT_2$) space.*

(ii.) *Let $X = \{a, b, c\}$, $\tau = P(X)$. Then X is not an rT_{min} space(resp.rT_{max}), but it is a δT_0 (respectively $\delta T_1, rT_2$) space.*

Remark 2.9.5:

rT_{min} (respectively rT_{max}) space need not be $rT_{\frac{1}{2}}$ space and vice-versa.

Example 2.9.2

(i.) *Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then (X, τ) is an rT_{min} and rT_{max}*

space, but it is not an $rT_{\frac{1}{2}}$ *space.*

(ii.) *Let* $X = \{a, b, c\}$, $\tau = P(X)$. *Then* X *is not an* rT_{min} *(respectively* rT_{max}*) space, but it is an* $rT_{\frac{1}{2}}$ *space.*

Remark 2.9.5:

rT_{min} (respectively rT_{max}) space need not be an r-door space and vice-versa.

Example 2.9.3

(i.) *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. (X, τ) *is an* rT_{min} *and* rT_{max} *space; but it is not an r-door space.*

(ii.) *Let* $X = \{a, b, c\}$, $\tau = P(X)$. *Then* X *is not an* rT_{min} *and* rT_{max} *space, but it is an r-door space.*

Remark 2.9.5:

rT_{min} and rT_{max} spaces need not be *submaximal regular space* and vice-versa.

Example 2.9.4

(i.) *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. *Then* (X, τ) *is an* rT_{min} *and* rT_{max} *space, but it is not a submaximal regular space.*

(ii.) *Let* $X = \{a, b, c\}$, $\tau = P(X)$. *Then* X *is not an* rT_{min} *and* rT_{max} *space, but it is a submaximal regular space.*

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CHAPTER 3

Various functions and their properties

3.1 Introduction

This chapter contains discussion about almost perfectly continuous function, somewhat r-continuous function, minimal & maximal r-continuous function, minimal-maximal and maximal-minimal r-continuous function, minimal & maximal r-irresolute function. Main topic of discussion is restriction, extension and composition of such functions. Throughout the chapter, X and Y denote topological spaces with topologies τ and σ respectively. Section 2 is on preliminary ideas. In section 3, properties of almost perfectly continuous functions are discussed. Somewhat r-continuous function and its properties are given in section 4. Section 5, is on minimal r-continuous and maximal r-continuous functions and their properties. Supra r-continuous functions and their properties are discussed in section 6.

3.2 Preliminary ideas

Definition 3.2.1

A function $f : X \to Y$ is said to be

1. *totally continuous [13], if inverse image of every open set of Y is clopen in X.*

2. *completely continuous [3], if inverse image of every open set of Y is regular open in X.*

3. *almost completely continuous [10], if inverse image of every regular open set of Y is regular open in X.*

4. *almost perfectly continuous [28], if inverse image of every regular open set of Y is clopen in X.*

5. *strongly continuous [16], if $f(Cl(A)) \subset f(A)$ for every $A \subset X$.*

6. *somewhat continuous [11], if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$, there exists an open set V in X such that $V \neq \phi$ and $V \subset f^{-1}(U)$.*

7. *cl-super continuous [14] ($\equiv$ clopen continuous [10]), if for each $x \in X$ and for each open set V containing $f(x)$, there exists a clopen set U containing x such that $f(U) \subset V$.*

8. *δ-continuous [24], if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subset V$.*

9. *almost continuous [27], if $f^{-1}(V)$ is an open set in X, for every regular open set V of Y.*

Definition 3.2.2

A space X is said to be r-connected, if X is not the union of two non empty disjoint regular open sets of X.

3.3 Properties of almost perfectly continuous functions

Theorem 3.3.1

A function $f : X \to Y$ is almost perfectly continuous if and only if the inverse image of every regular closed subset of Y is clopen in X.

Proof:

Let $f : X \to Y$ be almost perfectly continuous. Let F be a regular closed subset of Y. Then $Y - F$ is regular open in Y. Since f is almost perfectly continuous, $f^{-1}(Y - F) = X - f^{-1}(F)$ is clopen in X.

Conversely suppose that inverse image of every regular closed subset of Y is clopen in X. Let V be regular open in Y. Then $Y - V$ is regular closed in Y. Since inverse image of regular closed set is clopen in X, $f^{-1}(Y - V)$ is clopen in X. Hence $X - f^{-1}(V)$ is clopen in X. So f is almost perfectly continuous. $\square$

Theorem 3.3.2

Let $f : X \to Y$ be a function, where X and Y are topological spaces and X is finite. Then the following are equivalent.

(i.) *f is almost perfectly continuous.*

(ii.) *For each $x \in X$ and each regular open set V in Y with $f(x) \in V$, there exists a clopen set U in X such that $x \in U$ and $f(x) \in V$.*

Proof:

$(i) \Rightarrow (ii)$

Follows by taking $U = f^{-1}(V)$.

$(ii) \Rightarrow (i)$

Suppose (ii) holds. Let V be a regular open set in Y and $x \in f^{-1}(V)$. Then by (ii), there exist clopen set U_x in X such that $x \in U_x$ and $f(U_x) \subset V$. Hence $f^{-1}(V)$ is a clopen neighbourhood of each of its points. Since X finite, $f^{-1}(V)$ is clopen. So f is almost perfectly continuous. □

Theorem 3.3.3

Let $f : X \to Y$ be an almost perfectly continuous function from an r-connected space X onto any space Y. Then Y is an indiscrete space.

Proof:

Let $f : X \to Y$ be almost perfectly continuous. Suppose Y is not indiscrete. Let A be a proper non empty regular open subset of Y. Since f is almost perfectly continuous, $f^{-1}(A)$ is a proper non empty clopen subset of X. Since clopen sets are regular open, this is a contradiction to the fact that X is r-connected. So Y is an indiscrete space. □

Theorem 3.3.4

Every strongly continuous function is almost perfectly continuous.

Proof:

Let $f : X \to Y$ be strongly continuous. Let V be a regular open subset of Y. Since f is strongly continuous, $f^{-1}(V)$ is clopen in X. So f is almost perfectly continuous. □

Remark 3.3.4:

Converse of the above theorem need not be true.

Example 3.3.1

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}, \sigma = \{Y, \phi, \{b, c\}, \{c\}, \{a\}, \{a, c\}\}$.

Let $f : X \to Y$ *be the identity function. Then* f *is almost perfectly continuous, but not strongly continuous.*

Theorem 3.3.5

Every almost perfectly continuous function into a discrete space is strongly continuous.

Proof:

Suppose $f : X \to Y$ is almost perfectly continuous. Let A be a subset of Y. Then A is clopen and hence regular open. Since f is almost perfectly continuous, $f^{-1}(A)$ is clopen. So f is strongly continuous. □

corollary 3.3.6

Every almost perfectly continuous function into a finite T_1 *space is strongly continuous.*

Proof:

Result holds since every open set in a finite T_1 space is clopen. □

Theorem 3.3.7

Every almost perfectly continuous function is almost completely continuous.

Proof:

Proof follows from the result that clopen sets are regular open. □

Remark 3.3.7:

Converse of the above theorem need not be true.

Example 3.3.2

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{a\}, \{a, b\}\}, \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Let $f : X \to Y$ *be defined by* $f(a) = b$, $f(b) = a$, $f(c) = c$. *Then* f *is almost completely continuous, but not almost perfectly continuous.*

Theorem 3.3.8

Let $f : X \to Y$ *be almost completely continuous and* X *be locally indiscrete. Then* f *is almost perfectly continuous.*

Proof:

Let $V \subset Y$ be regular open in Y. Since f is almost completely continuous, $f^{-1}(V)$ is regular open and hence open in X. Since X is locally indiscrete, $f^{-1}(V)$ is closed. So f is almost perfectly continuous. $\square$

Theorem 3.3.9

Let $f : X \to Y$ *be almost perfectly continuous, where* Y *is locally indiscrete. Then* f *is completely continuous.*

Proof:

Proof follows from the result that 'open sets of locally indiscrete space are clopen and clopen sets are regular open'. $\square$

Theorem 3.3.10

If f is completely continuous and X is locally indiscrete, then f is almost perfectly continuous.

Proof:

Let $V \subset Y$ be regular open. Since f is completely continuous, $f^{-1}(V)$ is regular open. Since X is locally indiscrete, $f^{-1}(V)$ is clopen. Hence f is almost perfectly continuous. □

Theorem 3.3.11

Every totally continuous function is almost perfectly continuous.

Proof:

Let $f : X \to Y$ be totally continuous and $V \subset Y$ be regular open. Since f is totally continuous, $f^{-1}(V)$ is clopen. Hence f is almost perfectly continuous. □

Remark 3.3.11:

Converse of the above theorem need not be true.

Example 3.3.3

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}, \sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Let $f : X \to Y$ be the identity function. Then f is almost perfectly continuous, but not totally continuous.

Theorem 3.3.12

Let $f : X \to Y$ be almost perfectly continuous and Y be locally indiscrete. Then f is totally continuous.

Proof:

Let $V \subset Y$ be open. Since Y is locally indiscrete, V is clopen and hence regular open. Since f is almost perfectly continuous, $f^{-1}(V)$ is clopen. So f is totally continuous. $\square$

Remark 3.3.12:

The following diagram shows the relationship between various functions and almost perfectly continuous function.

$$\begin{array}{ccc}
\textit{Strongly continuous} & & \\
\downarrow & & \\
\textit{Totally continuous} & \rightarrow & \textit{Completely continuous} \\
\downarrow & & \downarrow \\
\textit{Almost perfectly continuous} & \rightarrow & \text{Almost completely continuous}
\end{array}$$

Theorem 3.3.13

Composition of two almost perfectly continuous functions is almost perfectly continuous.

Proof:

Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be almost perfectly continuous. Let $V \subset Z$ be regular open. Since g is almost perfectly continuous, $g^{-1}(V)$ is clopen and hence regular open. Since f is almost perfectly continuous, $f^{-1}(g^{-1}(V))$ is clopen in X. So $g \circ f$ is almost perfectly continuous. $\square$

Theorem 3.3.14

Composition of almost perfectly continuous function and almost completely continuous function is almost perfectly continuous.

Proof:

Let $f : X \to Y$ be almost perfectly continuous and $g : Y \to Z$ be almost completely continuous. Let $V \subset Z$ be regular open. Since g is almost completely continuous, $g^{-1}(V)$ is regular open in Y. Since f is almost perfectly continuous, $f^{-1}(g^{-1}(V))$ is clopen in X. So $g \circ f$ is almost perfectly continuous.

□

Theorem 3.3.15

Composition of almost perfectly continuous function and completely continuous function is totally continuous.

Proof:

Let $f : X \to Y$ be almost perfectly continuous and $g : Y \to Z$ be completely continuous. Let $V \subset Z$ be open. Since g is completely continuous, $g^{-1}(V)$ is regular open in Y. Since f is almost perfectly continuous, $f^{-1}(g^{-1}(V))$ is clopen in X. So $g \circ f$ is totally continuous. □

Theorem 3.3.16

Let $f : X \to Y$ be almost perfectly continuous and $g : Y \to Z$ be any function. Then $g \circ f : X \to Z$ is almost perfectly continuous if and only if g is almost completely continuous.

Proof:

Suppose $g \circ f : X \to Z$ is almost perfectly continuous. Let $V \subset Z$ be regular open. Then $(g \circ f)^{-1}(V)$ is clopen in X. Since f is almost perfectly continuous, this is possible only if $g^{-1}(V)$ is regular open. So g is almost completely continuous. $g \circ f$ almost perfectly continuous follows from theorem 3.3.14.

$\square$

Theorem 3.3.17

If a function $f : X \to \prod Y_\lambda$ is almost perfectly continuous, then $\pi_\lambda \circ f : X \to Y_\lambda$ is almost perfectly continuous for each $\lambda \in \Lambda$, where π_λ is the projection of $\prod Y_\lambda$ onto Y_λ.

Proof:

For each $\lambda \in \Lambda$, suppose V_λ is regular open in Y_λ. Then $\pi_\lambda^{-1}(V_\lambda)$ is regular open in $\prod Y_\lambda$. Since $f : X \to \prod Y_\lambda$ is almost perfectly continuous, $f^{-1}(\pi_\lambda^{-1}(V_\lambda))$ is clopen in X. Hence $\pi_\lambda \circ f : X \to Y_\lambda$ is almost perfectly continuous for each $\lambda \in \Lambda$. $\square$

Theorem 3.3.18

Restriction of an almost perfectly continuous function onto a clopen set is almost perfectly continuous.

Proof:

Let $f : X \to Y$ be almost perfectly continuous and A is a clopen subset of X. Consider $f/A : A \to Y$. Let V be a regular open subset of Y. Since f is almost perfectly continuous, $f^{-1}(V)$ is clopen in X. Since A is clopen, $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is clopen in A. So f/A is almost perfectly continuous. $\square$

3.4 Somewhat r-continuous function and its properties

Definition 3.4.1 *Let X and Y be any two topological spaces. A function $f : X \to Y$ is said to be somewhat r-continuous, if $U \in \sigma$ and $f^{-1}(U) \neq \phi$, then there exists a regular open set V in X such that $V \neq \phi$ and $V \subset f^{-1}(U)$.*

Example 3.4.1

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}\}, \sigma = \{X, \phi, \{b, c\}\}$.

Define $f : X \to Y$ by $f(a) = a,\ f(b) = c,\ f(c) = b$. Then f is somewhat r-continuous.

Definition 3.4.2

Let M be a subset of a topological space (X, τ). Then M be said to be r-dense in X, if there is no regular closed set C in X such that $M \subset C \subset X$.

Theorem 3.4.1

Let $f : X \to Y$ be an injective function. Then the following are equivalent.

- *(i.)* *f is somewhat r-continuous.*
- *(ii.)* *If C is a closed subset of Y such that $f^{-1}(C) \neq \phi$, then, there is a proper regular closed subset D of X such that $D \supset f^{-1}(C)$.*
- *(iii.)* *If M is an r-dense subset of X, then $f(M)$ is a dense subset of Y.*

Proof:

$(i) \Rightarrow (ii)$

Let C be a closed subset of Y such that $f^{-1}(C) \neq \phi$. Then $Y - C$ is open in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$. Since f is somewhat r-continuous, there exists regular open set V such that $V \subset X - f^{-1}(C)$. This implies $f^{-1}(C) \subset X - V$. Since V is regular open, $X - V = D$ is regular closed.

$(ii) \Rightarrow (iii)$

Let M be an r-dense subset of X. Suppose $f(M)$ is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq \phi$. Hence by (ii), there exists proper regular closed set D such that $D \supset f^{-1}(C)$. That is $M \subset f^{-1}(C) \subset D \subset X$. This contradicts the fact that M is r-dense in X. So $f(M)$ is dense in Y.

$(iii) \Rightarrow (ii)$

Suppose (ii) is not true. Then for closed set C with $f^{-1}(C) \neq \phi$, there is no proper regular closed set D in X such that $f^{-1}(C) \subset D$. This means $f^{-1}(C)$ is r-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, a contradiction to the choice of C. So (ii) is true.

$(ii) \Rightarrow (i)$

Let U be an open set in Y and $f^{-1}(U) \neq \phi$. Then $Y - U$ is closed in Y and $f^{-1}(Y - U) = X - f^{-1}(U) \neq \phi$. So by (ii), there exists a proper regular closed subset D of X such that $D \supset f^{-1}(Y - U) = X - f^{-1}(U)$. That is $X - D \subset f^{-1}(U)$ and $X - D$ is a non empty regular open subset. So f is somewhat r-continuous. □

Definition 3.4.3

If X is a set and τ and σ are topologies for X, then τ is said to be r-weakly equivalent to σ, if for every non empty U in τ, there is a non empty regular open set V in σ such that

$V \subset U$ and for every non empty set U in σ, there is a non empty regular open set V in τ such that $V \subset U$.

Theorem 3.4.2

Let $f : X \to Y$ be a somewhat r-continuous function. Let σ^* be a topology for Y which is weakly equivalent to σ. Then $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat r-continuous.

Proof:

Let U be an open set in (Y, σ^*) such that $f^{-1}(U) \neq \phi$. Then $U \neq \phi$. Since σ and σ^* are weakly equivalent, there exists an open set W in (Y, σ) such that $W \neq \phi$ and $W \subset U$. Then $f^{-1}(W) \neq \phi$. Since f is somewhat r-continuous, there exists regular open set $V \neq \phi$ such that $V \subset f^{-1}(W)$. Then $V \subset f^{-1}(W) \subset f^{-1}(U)$. So $f : (X, \tau) \to (Y, \sigma^*)$ is somewhat r-contnuous. □

Theorem 3.4.3

Every somewhat r-continuous function is somewhat continuous.

Proof:

Proof follows from the result that 'regular open sets are open'. □

Remark 3.4.3:

Converse of the above theorem does not hold.

Example 3.4.2 *Let $X = \{a, b, c, d\}, Y = \{p, q, r\}$*

$\tau = \{X, \phi, \{a, c\}, \{d\}, \{c\}, \{c, d\}, \{a, c, d\}\}$.

$\sigma = \{Y, \phi, \{r\}, \{q\}, \{r, q\}\}$. *Define $f : X \to Y$ by $f(a) = f(d) = q, f(c) = f(b) = r$. Then f is somewhat continuous, but not somewhat r-continuous.*

Theorem 3.4.4 *If $f : X \to Y$ is somewhat continuous and X is locally indiscrete, then f is somewhat r-continuous.*

Proof:

The proof follows from the result that open sets in a locally indiscrete space are clopen and clopen sets are regular open. □

Theorem 3.4.5

Every cl-super continuous function is somewhat r-continuous.

Proof:

The proof follows from the result that clopen sets are regular open. □

Remark 3.4.5:

Converse of the above theorem does not hold.

Example 3.4.3

Let $X = \{a, b, c, d\}, Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}, \{a, b, d\}\}$.

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{a,\ b\}\}$. *Let $f : X \to Y$ be the identity function. Then f is somewhat r-continuous, but not cl-supercontinuous.*

Theorem 3.4.6

Let $f : X \to Y$ be somewhat r-continuous and X be locally indiscrete. Then f is cl-super continuous.

Proof:

The proof follows from the result that, regular open set is open and open set in a locally indiscrete space is clopen. □

Theorem 3.4.7

Every completely continuous function is somewhat r-continuous.

Remark 3.4.7:

Converse of the above theorem does not hold.

Example 3.4.4

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}\}, \sigma = \{Y, \phi, \{a, b\}\}$.

Let $f : X \to Y$ *be the identity function. Then* f *is somewhat r-continuous, but not completely continuous.*

Theorem 3.4.8

If X *is a discrete space and* $f : X \to Y$ *is somewhat r-continuous, then* f *is completely continuous.*

Proof:

The proof follows from the result that finite union of regular open sets in a discrete space is regular open. □

corollary 3.4.9

If X *is finite,* T_1 *and* $f : X \to Y$ *is somewhat r-continuous, then* f *is completely continuous.*

Proof:

Proof follows from the result that finite union of regular open sets in a finite T_1 space is regular open. □

Theorem 3.4.10

Every somewhat r-continuous function is $\delta-continuous$.

Proof:

Let $f : X \to Y$ be somewhat r-continuous. Let V be non empty regualr open set in Y. Then it is open. Since f is somewhat r-continuous, there exists a regular open set U such that $f(U) \subset V$. So f is $\delta - continuous$. □

Remark 3.4.10:

Converse of the above theorem does not hold.

Example 3.4.5

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{c\}\}, \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. *Define* $f : X \to Y$ *by* $f(a) = b, f(b) = c, f(c) = a$. *Then* f *is* $\delta - continuous$, *but not somewhat r-continuous.*

Theorem 3.4.11 *If $f : X \to Y$ is δ-continuous and Y is locally indiscrete, then f is somewhat r-continuous.*

Proof:

Let V be open in Y. Since Y is locally indiscrete, V is clopen and so regular open. Since f is $\delta - continuous$, there exists regular open set U such that $f(U) \subset V$. So f is somewhat r-continuous. □

Theorem 3.4.12

If $f : X \to Y$ is almost completely continuous and Y is locally indiscrete, then f is somewhat r-continuous.

Proof:

Let V be open in Y. Since Y is locally indiscrete, V is clopen and so regular open. Since f is almost completely continuous, $f^{-1}(V) = U$ is regular open. So f is somewhat r-continuous.

□ □

Theorem 3.4.13

If $f : X \to Y$ is somewhat r-continuous and X is a discrete space, then f is almost completely continuous.

Proof:

Proof follows from the result that finite union of regular open sets in a discrete space is regular open. □

corollary 3.4.14

If $f : X \to Y$ is somewhat r-continuous, X is finite and T_1, then f is almost completely continuous.

Proof:

Proof follows from the result that finite union of regular open sets in a finite T_1 space is regular open. □

Theorem 3.4.15

Let $f : X \to Y$ be somewhat continuous and τ^* be a topology for X which is r-weakly equivalent to τ. Then the function $f : (X, \tau^*) \to (Y, \sigma)$ is somewhat r-continuous.

Proof:

Let U be any open set in (Y, σ) such that $f^{-1}(U)$ is non empty. Since f is somewhat continuous, there exists a non empty open set V in X such that $V \subset f^{-1}(U)$. Since τ is r-equivalent to τ^*, there exists a non empty regular open set V_1 in (X, τ^*) such that $V_1 \subset V \subset f^{-1}(U)$. So f is somewhat r-continuous. □

Remark 3.4.15:

The following diagram shows the relationship between various functions and somewhat r-continuous function.

		$cl - super \quad continuous$		
		↓		
completely continuous	→	somewhat r- continuous	→	somewhat continuous
↓		↓		
almost completely continuous	→	$\delta - continuous$		

Theorem 3.4.16

Composition of a continuous function and a somewhat r-continuous function is somewhat r-continuous.

Proof:

Consider the continuous function $g : Y \to Z$ and the somewhat r-continuous function $f : X \to Y$. Let $V \subset Z$ be open. Since g is continuous, $g^{-1}(V)$ is open in Y. Since f is somewhat r-continuous, there exists regular open set U such that $U \subset f^{-1}(g^{-1}(V))$. Hence $g \circ f$ is somewhat r-continuous. □

Theorem 3.4.17

Composition of a somewhat r-continuous function and a continuous function is somewhat *r-continuous.*

Proof:

Consider the somewhat r- continuous function $g : Y \to Z$ and the continuous function $f : X \to Y$. Let $V \subset Z$ be open. Since g is somewhat r-continuous, there exists regular open set U such that $U \subset g^{-1}(V)$. Then U is open and by continuity of $f, f^{-1}(U)$ is open. Now $f^{-1}(U) \subset f^{-1}(g^{-1}(V))$. So $g \circ f$ is continuous. $\square$

Theorem 3.4.18

Let X and Y be any two topological spaces. Let A be a regular open set of X and $f : (A, \tau/A) \to (Y, \sigma)$ be somewhat r-continuous such that $f(A)$ is dense in Y. Then any extension F of f is somewhat r-continuous.

Proof:

Let U be any open set in Y such that $F^{-1}(U) \neq \phi$. Since $f(A)$ is dense in $Y, U \cap f(A) \neq \phi$. So $F^{-1}(U) \cap A \neq \phi$. Hence $f^{-1}(U) \cap A \neq \phi$. Since f is somewhat r-continuous, there exists a regular open set V such that $V \subset f^{-1}(U) \subset F^{-1}(U)$. Hence F is somewhat r-continuous. $\square$

Theorem 3.4.19

Let X and Y be any two topological spaces. If $Z = A \cap B$ where A and B are regular open subsets of X and if $f : Z \to Y$ is a function such that f/A and f/B are somewhat r-continuous, then f is somewhat r-continuous.

Proof:

Let V be any open set in Y such that $f^{-1}(V) \neq \phi$. Then either $(f/A)^{-1}(V) \neq \phi$ or

$(f/B)^{-1}(V) \neq \phi$ or both.

Case (i): $(f/A)^{-1}(V) \neq \phi$.

Since f/A is somewhat r-continuous, there exists a non empty regular open set V_1 in A such that $V_1 \subset (f/A)^{-1}(V) \subset f^{-1}(V)$. Since V_1 is regular open in A and A is regular open in X, V_1 is regular open in X. So f is somewhat r-continuous.

Case (ii): $(f/B)^{-1}(V) \neq \phi$.

This can be proved by using the same argument as in (i).

Case (iii):$(f/A)^{-1}(V) \neq \phi$ and $(f/B)^{-1}(V) \neq \phi$

The proof follows from the proofs of case(i) and case(ii). □

3.5 Minimal r-continuous function, maximal r-continuous function and their properties.

Definition 3.5.1

Let X and Y be topological spaces. A function $f : X \to Y$ is called

1. *minimal r-continuous, if $f^{-1}(M)$ is a regular open set in X, for every minimal regular open set M in Y.*

2. *maximal r-continuous, if $f^{-1}(M)$ is a regular open set in X, for every maximal regular open set M in Y.*

3. *minimal r-irresolute, if $f^{-1}(M)$ is a minimal regular open set in X, for every minimal regular open set M in Y.*

4. *maximal r-irresolute, if $f^{-1}(M)$ is a maximal regular open set in X, for every max-*

imal regular open set M in Y.

5. *minimal - maximal r-continuous, if $f^{-1}(M)$ is a maximal regular open set in X, for every minimal regular open set M in Y.*

6. *maximal- minimal r-continuous, if $f^{-1}(M)$ is a minimal regular open set in X, for every maximal regular open set M in Y.*

Theorem 3.5.1

Let X and Y be topological spaces. A function $f : X \rightarrow Y$ is minimal r-continuous if and only if the inverse image of each maximal regular closed set in Y is a regular closed set in X.

Proof:

Proof holds from the definition of minimal r-continuous function and the result that a set is minimal regular open if and only if it is maximal regular closed. □

Theorem 3.5.2

Let X and Y be topological spaces. A function $f : X \rightarrow Y$ is maximal r-continuous if and only if the inverse image of each minimal regular closed set in Y is a regular closed set in X.

Proof:

Proof holds from the definition of maximal r-continuous function and the result that a set is maximal regular open if and only if it is minimal regular closed. □

Theorem 3.5.3

Let X and Y be topological spaces. A function $f : X \rightarrow Y$ is minimal r-irresolute if and *only if the inverse image of each maximal regular closed set in Y is a maximal regular closed set in X.*

Proof:

Proof holds from the definition of minimal r-irresolute function and the result that a set is minimal regular open if and only if it is maximal regular closed. □

Theorem 3.5.4

Let X and Y be topological spaces. A function $f : X \to Y$ is maximal r-irresolute if and only if the inverse image of each minimal regular closed set in Y is a minimal regular closed set in X.

Proof:

Proof holds from the definition of maximal r-irresolute function and the result that a set is maximal regular open if and only if it is minimal regular closed. □

Theorem 3.5.5

Let X and Y be topological spaces. A function $f : X \to Y$ is maximal-minimal r-continuous if and only if the inverse image of each minimal regular closed set in Y is a maximal regular closed set in X.

Proof:

Proof holds from the definition of maximal-minimal r-continuous function and the result that a set is minimal regular open if and only if it is maximal regular closed and is maximal regular open if and only if it is minimal regular closed. □

Theorem 3.5.6

Let X and Y be topological spaces. A function $f : X \to Y$ is minimal-maximal r-continuous if and only if the inverse image of each maximal regular closed set in Y is a minimal regular closed set in X.

Proof:

Proof holds from the definition of minimal - maximal r-continuous function and the result that a set is minimal regular open if and only if it is maximal regular closed and is maximal regular open if and only if it is minimal regular closed. □

Theorem 3.5.7

Every almost completely continuous function is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open and $f : X \to Y$ almost completely continuous. Then as minimal regular open set is regular open, V is regular open. f is almost completely continuous implies $f^{-1}(V)$ is regular open. Hence f is minimal r-continuous. □

Remark 3.5.7:

Converse of the above theorem need not be true.

Theorem 3.5.8

If Y is an rT_{min} space and $f : X \to Y$ is a minimal r-continuous onto function, then f is almost completely continuous.

Proof:

Let $V \subset Y$ be regular open. Since Y is an rT_{min} space, V is minimal regular open in Y.

$f : X \to Y$ is a minimal r-continuous onto function implies $f^{-1}(V)$ is regular open. Hence f is almost completely continuous. □

Theorem 3.5.9

Every almost completely continuous function is maximal r-continuous.

Proof:

Let $V \subset Y$ be maximal regular open and $f : X \to Y$ almost completely continuous. Then as maximal regular open set is regular open, V is regular open. f is almost completely continuous implies $f^{-1}(V)$ is regular open. Hence f is maximal r-continuous. □

Remark 3.5.9:

Converse of the above theorem need not be true.

Theorem 3.5.10

If Y is an rT_{max} space and $f : X \to Y$ is a maximal r-continuous onto function, then f is almost completely continuous.

Proof:

Let $V \subset Y$ be regular open. Since Y is an rT_{max} space, V is maximal regular open in Y. $f : X \to Y$ is a maximal r-continuous onto function implies $f^{-1}(V)$ is regular open. Hence f is almost completely continuous. □

Theorem 3.5.11

Every strongly continuous function is minimal r-continuous.

Proof:

Proof follows from the fact that minimal regular open sets are open and clopen sets are regular open. □

Remark 3.5.11:

Converse of the above theorem need not be true.

Example 3.5.1

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}$.

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. *Define* $f : X \to Y$ *by* $f(a) = a, f(b) = c, f(c) = b$. *Then* f *is minimal r-continuous, but not strongly continuous.*

Theorem 3.5.12

Let $f : X \to Y$ *be minimal r-continuous, where* X *is locally indiscrete,* Y *is discrete and* rT_{min}*, then* f *is strongly continuous.*

Proof:

Let $A \subset Y$. Since Y is a discrete space, A is clopen and so regular open. Y is rT_{min} implies A is minimal regular open. Since $f : X \to Y$ is minimal r-continuous, $f^{-1}(A)$ is regular open. Since X is locally indiscrete, $f^{-1}(A)$ is clopen. Hence f is strongly continuous. □

Theorem 3.5.13

If $f : X \to Y$ *is minimal r-continuous, where* Y *is an* rT_{min} *space, then* f *is almost continuous.*

Proof:

Let $V \subset Y$ be regular open. Since Y is an rT_{min} space, V is minimal regular open.

$f : X \to Y$ is minimal r-continuous implies that $f^{-1}(V)$ is regular open and so open. Hence f is almost continuous. □

Theorem 3.5.14

If $f : X \rightarrow Y$ is almost continuous, where X is locally indiscrete, then f is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Then V is regular open. $f : X \rightarrow Y$ is almost continuous implies $f^{-1}(V)$ is open . Since X is locally indiscrete, V is clopen and so regular open. Hence f is minimal r-continuous. □

Theorem 3.5.15

Every completely continuous function is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Then V is open. $f : X \rightarrow Y$ completely continuous implies that $f^{-1}(V)$ is regular open. Hence f is minimal r- continuous. □

Remark 3.5.15:

Converse of the above theorem need not be true.

Example 3.5.2

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}$.

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. Define $f : X \rightarrow Y$ by $f(a) = a, f(b) = c, f(c) = b$. Then f is minimal r-continuous, but not completely continuous.

Theorem 3.5.16

Every minimal r-continuous function onto a locally indiscrete rT_{min} space is completely continuous.

Proof:

Let $V \subset Y$ be open. Since Y is locally indiscrete rT_{min} space, V is minimal regular open. Since $f : X \to Y$ is minimal r- continuous, $f^{-1}(V)$ is regular open. Hence f is completely continuous. □

Theorem 3.5.17

Every completely continuous function is maximal r-continuous.

Proof:

Let $V \subset Y$ be maximal regular open. Then V is open. $f : X \to Y$ is completely continuous implies that $f^{-1}(V)$ is regular open. Hence f is maximal r- continuous. □

Remark 3.5.17:

Converse of the above theorem need not be true.

Example 3.5.3

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}$.

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. *Define* $f : X \to Y$ *by* $f(a) = a, f(b) = c, f(c) = b$. *Then* f *is maximal r-continuous, but not completely continuous.*

Theorem 3.5.18

Every almost perfectly continuous function is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Then V is regular open. $f : X \to Y$ is almost perfectly continuous implies that $f^{-1}(V)$ is clopen and hence regular open. Hence f is minimal r- continuous. □

Remark 3.5.18:

Converse of the above theorem need not be true.

Example 3.5.4

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}$.

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}\}$. *Define* $f : X \to Y$ *by* $f(a) = a, f(b) = c, f(c) = b$. *Then* f *is minimal r-continuous, but not almost perfectly continuous.*

Theorem 3.5.19

If $f : X \to Y$ *is minimal r-continuous, where* X *is locally indiscrete and* Y *is* rT_{min}, *then* f *is almost perfectly continuous.*

Proof:

Let $V \subset Y$ be regular open. Since Y is rT_{min}, V is minimal regular open. $f : X \to Y$ is minimal r-continuous implies that $f^{-1}(V)$ is regular open. X is locally indiscrete implies that $f^{-1}(V)$ is clopen. Hence f is almost perfectly continuous. □

Theorem 3.5.20

If $f : X \to Y$ *is maximal r-continuous, where* X *is locally indiscrete and* Y *is* rT_{max}, *then* f *is almost perfectly continuous.*

Proof:

Let $V \subset Y$ be regular open. Since Y is rT_{max}, V is maximal regular open. $f : X \to Y$

is maximal r-continuous implies $f^{-1}(V)$ is regular open. X is locally indiscrete implies $f^{-1}(V)$ is clopen. Hence f is almost perfectly continuous. □

Theorem 3.5.21

Every totally continuous function is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Then V is open. $f : X \to Y$ is totally continuous implies that $f^{-1}(V)$ is clopen and so regular open. Hence f is minimal r-continuous. □

Remark 3.5.21:

Converse of the above theorem need not be true.

Example 3.5.5

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}, \sigma = \{X, \phi, \{a, c\}, \{a\}, \{c\}\}$.

Define $f : X \to Y$ *by* $f(a) = c, f(b) = a, f(c) = b$. *Then* f *is minimal r-continuous, but not totally continuous.*

Theorem 3.5.22

If $f : X \to Y$ *is minimal r-continuous, where* X *and* Y *are locally indiscrete and* Y *is* rT_{min}, *then* f *is totally continuous.*

Proof:

Let $V \subset Y$ be open. Since Y is rT_{min} and locally indiscrete, V is clopen and minimal regular open. $f : X \to Y$ is minimal r-continuous implies that $f^{-1}(V)$ is regular open. X is locally indiscrete implies that $f^{-1}(V)$ is clopen. Hence f is totally continuous. □

Theorem 3.5.23

Every totally continuous function is maximal r-continuous.

Proof:

Let $V \subset Y$ be maximal regular open. Then V is open. $f : X \to Y$ is totally continuous implies that $f^{-1}(V)$ is clopen and so regular open. Hence f is maximal r-continuous. □

Remark 3.5.23:

Converse of the above theorem need not be true.

Example 3.5.6

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}\}, \sigma = \{X, \phi, \{a, c\}, \{a\}, \{c\}\}$.

Define $f : X \to Y$ *by* $f(a) = c, f(b) = a, f(c) = b$. *Then* f *is maximal r-continuous, but not totally continuous.*

Theorem 3.5.24

If $f : X \to Y$ *is maximal r-continuous, where* X *and* Y *are locally indiscrete and* Y *is* rT_{max}*, then* f *is totally continuous.*

Proof:

Let $V \subset Y$ *be open. Since* Y *is* rT_{max} *and locally indiscrete,* V *is clopen and maximal regular open.* $f : X \to Y$ *is maximal r-continuous implies that* $f^{-1}(V)$ *is regular open.* X *is locally indiscrete implies that* $f^{-1}(V)$ *is clopen. Hence* f *is totally continuous.* □

Theorem 3.5.25

Every minimal r-irresolute function is minimal r-continuous.

Proof:

Proof follows from the definition of minimal r-irresolute function, minimal r-continuous function and the property that minimal regular open set is regular open. □

Remark 3.5.25:

Converse of the above theorem need not be true.

Example 3.5.7

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}, \{a, c\}, \{b, c\}, \{c\}\}$

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{b\}, \{a, b\}, \{b, c\}\}$.

Define $f : X \to Y$ *by* $f(a) = b, f(b) = b, f(c) = c$. *Then* f *is minimal r-continuous, but not minimal r-irresolute.*

Theorem 3.5.26

Let $f : X \to Y$ *be minimal r-continuous(respectively maximal r-continuous) where* X *is an* rT_{min} *(respectively* rT_{max}*) space. Then* f *is minimal r-irresolute (respectively maximal r-irresolute).*

Proof:

Proof holds from the definition of rT_{min} space (respectively rT_{max} space) and minimal r-irresolute function(respectively maximal r-irresolute function). □

Theorem 3.5.27

Every minimal r-irresolute function onto an rT_{min} *space is almost completely continuous.*

Proof:

Let $V \subset Y$ be regular open. If Y is rT_{min}, then V is minimal regular open. If $f : X \to Y$ is minimal r-irresolute, then $f^{-1}(V)$ is minimal regular open and hence regular open. Hence f is almost completely continuous. □

Theorem 3.5.28

Every maximal r-irresolute function onto an rT_{max} space is almost completely continuous.

Proof:

Let $V \subset Y$ be regular open. If Y is rT_{max}, then V is maximal regular open. If $f : X \to Y$ is maximal r-irresolute, then $f^{-1}(V)$ is maximal regular open and hence regular open. Hence f is almost completely continuous. □

Remark 3.5.28:

Converse of the above theorem need not be true.

Example 3.5.8

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}, \{a\}, \{b\}, \{a, c\}, \{b, c\}, \{c\}\}$

$\sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{b\}, \{a, b\}, \{b, c\}\}$.

Define $f : X \to Y$ *by* $f(a) = b, f(b) = b, f(c) = c$. *Then* f *is almost completely continuous, but not minimal r-irresolute.*

Theorem 3.5.29

If X *is an* rT_{min} *space, then every almost completely continuous function* $f : X \to Y$ *is minimal r-irresolute.*

Proof:

Let $V \subset Y$ be minimal regular open. Then V is regular open. If $f : X \to Y$ is almost completely continuous, then $f^{-1}(V)$ is regular open. If X is an rT_{min} space, then $f^{-1}(V)$ is minimal regular open. Hence f is minimal r-irresolute. □

Theorem 3.5.30

If X is an rT_{max} space, then every almost completely continuous function $f : X \to Y$ is maximal r-irresolute.

Proof:

Let $V \subset Y$ be maximal regular open. Then V is regular open. If $f : X \to Y$ is almost completely continuous, then $f^{-1}(V)$ is regular open. If X is an rT_{max} space, then $f^{-1}(V)$ maximal regular open. Hence f is maximal r-irresolute. □

Theorem 3.5.31

Every minimal - maximal r-continuous function is minimal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Since $f : X \to Y$ is minimal-maximal r-continuous, $f^{-1}(V)$ is maximal regular open and hence regular open. Hence f is minimal r-continuous.

□ □

Theorem 3.5.32

Every minimal r-continuous function from an rT_{max} space is minimal- maximal r-continuous.

Proof:

Let $V \subset Y$ be minimal regular open. Since $f : X \to Y$ is minimal r-continuous, $f^{-1}(V)$

is regular open. Since X is an rT_{max} space, $f^{-1}(V)$ is maximal regular open. Hence f is minimal-maximal r-continuous. □

Theorem 3.5.33

Every maximal- minimal r-continuous function is maximal r-continuous.

Proof:

Let $V \subset Y$ be maximal regular open. Since $f : X \to Y$ is maximal- minimal r-continuous, $f^{-1}(V)$ is minimal regular open and hence regular open. Hence f is maximal r-continuous.

□ □

Theorem 3.5.34

Every maximal r- continuous function from an rT_{min} space is maximal - minimal r-continuous.

Proof:

Let $V \subset Y$ be maximal regular open. Since $f : X \to Y$ is maximal r-continuous, $f^{-1}(V)$ is regular open. Since X is an rT_{min} space, $f^{-1}(V)$ is minimal regular open. Hence f is maximal-minimal r-continuous.

□ □

Theorem 3.5.35

Composition of an almost completely continuous function and a minimal r- continuous function is minimal r- continuous.

Proof:

Let $f : X \to Y$ be almost completely continuous and $g : Y \to Z$ be minimal r-continuous. Let $V \subset Z$ be minimal regular open. Since g is minimal r-continuous, $g^{-1}(V)$ is regular open in Y. Since f is almost completely continuous, $f^{-1}(g^{-1}(V))$ is regular open in X. So $g \circ f$ is minimal r-continuous. □

Theorem 3.5.36

Composition of an almost completely continuous function and a maximal r- continuous function is maximal r- continuous.

Proof:

Let $f : X \to Y$ be almost completely continuous and $g : Y \to Z$ be maximal r-continuous. Let $V \subset Z$ be maximal regular open. Since g is maximal r-continuous, $g^{-1}(V)$ is regular open in Y. Since f is almost completely continuous, $f^{-1}(g^{-1}(V))$ is regular open in X. So $g \circ f$ is maximal r-continuous. □

Theorem 3.5.37

Composition of maximal r-irresolute functions is maximal r- irresolute.

Proof:

Let $f : X \to Y$ and $g : Y \to Z$ be maximal r-irresolutes. Let $V \subset Z$ be maximal regular open. Since g is maximal r-irresolute, $g^{-1}(V)$ is maximal regular open in Y. Since f is maximal r-irresolute, $f^{-1}(g^{-1}(V))$ is maximal regular open in X. So $g \circ f$ is maximal r-irresolute. □

Remark 3.5.37:

Composition of minimal - maximal r-continuous functions need not be minimal - maximal r-continuous.

Example 3.5.9

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}, \{b\}, \{a\}, \{a, b\}\}$ *and*

$\sigma = \{Y, \phi, \{a, b\}, \{a\}, \{b\}, \{b, c\}\}$.

Suppose $f : X \to Y$ *is defined by* $f(a) = b, f(b) = a, f(c) = c$. *Then* f *is minimal-maximal r-continuous, but* $f \circ f$ *is not minimal-maximal r-continuous.*

Theorem 3.5.38

If $f : X \to Y$ *and* $g : Y \to Z$ *are minimal- maximal r-continuous and if* Y *is an* rT_{min} *space, then* $g \circ f : X \to Z$ *is minimal - maximal r- continuous..*

Proof:

Let $V \subset Z$ be minimal regular open. Since $g : Y \to Z$ is minimal- maximal r-continuous, $g^{-1}(V)$ is maximal regular open. Y is an rT_{min} space implies that $g^{-1}(V)$ is minimal regular open. Also since $f : X \to Y$ is minimal-maximal r-continuous $f^{-1}(g^{-1}(V))$ is maximal regular open. Hence $g \circ f : X \to Z$ is minimal- maximal r-continuous. □

Theorem 3.5.39

If $f : X \to Y$ *is maximal r-irresolute and* $g : Y \to Z$ *is minimal - maximal r-continuous, then* $g \circ f : X \to Z$ *is minimal- maximal r- continuous.*

Proof:

Let $V \subset Z$ be minimal regular open. Since $g : Y \to Z$ is minimal-maximal r-continuous, $g^{-1}(V)$ is maximal regular open. Since $f : X \to Y$ is maximal r-irresolute, $f^{-1}(g^{-1}(V))$ is maximal regular open. Hence $g \circ f : X \to Z$ is minimal-maximal r-continuous. □

Theorem 3.5.40

If $f : X \to Y$ is maximal r-continuous and $g : Y \to Z$ is minimal- maximal r-continuous, then $g \circ f : X \to Z$ is minimal r- continuous.

Proof:

Let $V \subset Z$ be minimal regular open. Since $g : Y \to Z$ is minimal- maximal r-continuous, $g^{-1}(V)$ is maximal regular open. Since $f : X \to Y$ is maximal r-continuous, $f^{-1}(g^{-1}(V))$ is regular open. Hence $g \circ f : X \to Z$ is minimal r-continuous. □

Remark 3.5.40:

Composition of maximal- minimal r-continuous functions need not be maximal- minimal r-continuous.

Example 3.5.10

Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}, \{a\}, \{b\}, \{a, b\}\}$ and

$\sigma = \{Y, \phi, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$.

Suppose $f : X \to Y$ is defined by $f(a) = b, f(b) = a, f(c) = c$. Then f is maximal-minimal r-continuous, but $f \circ f$ is not maximal- minimal r-continuous.

Theorem 3.5.41

Composition of a minimal r-irresolute function and a maximal- minimal r-continuous function is maximal - minimal r- continuous.

Proof:

Let $f : X \to Y$ be minimal r-irresolute and $g : Y \to Z$ be maximal- minimal r-continuous. Let $V \subset Z$ be maximal regular open. Since g is maximal-minimal r-continuous,

$g^{-1}(V)$ is minimal regular open in Y. Since f is minimal r-irresolute, $f^{-1}(g^{-1}(V))$ is minimal regular open in X. So $g \circ f$ is maximal- minimal r-continuous. □

Theorem 3.5.42

Composition of a minimal r-continuous function and a maximal- minimal r-continuous function is maximal r- continuous.

Proof:

Let $f : X \to Y$ be minimal r-continuous and $g : Y \to Z$ be maximal- minimal r-continuous. Let $V \subset Z$ be maximal regular open. Since g is maximal- minimal r-continuous, $g^{-1}(V)$ is minimal regular open in Y. Since f is minimal r-continuous, $f^{-1}(g^{-1}(V))$ is regular open in X. So $g \circ f$ is maximal r-continuous. □

Theorem 3.5.43

If $f : X \to Y$ and $g : Y \to Z$ are maximal- minimal r-continuous and if Y is an rT_{max} space, then $g \circ f : X \to Z$ is maximal- minimal r- continuous.

Proof:

Let $V \subset Z$ be maximal regular open. Since g is maximal-minimal r-continuous, $g^{-1}(V)$ is minimal regular open in Y. Since Y is an rT_{max} space, $g^{-1}(V)$ is maximal regular open in Y. Since f is maximal- minimal r-continuous, $f^{-1}(g^{-1}(V))$ is minimal regular open in X. So $g \circ f$ is maximal- minimal r-continuous. □

Theorem 3.5.44

Let X and Y be topological spaces and A be a non empty regular open subset of X. If

$f : X \to Y$ is minimal r-continuous, then the restriction function $f/A : A \to Y$ is minimal r-continuous.

Proof:

Let V be a minimal regular open subset of Y. Since f is minimal r-continuous, $f^{-1}(V)$ is regular open in X. Since A is regular open, $(f/A)^{-1}(V) = A \cap f^{-1}(V)$ is regular open in A. So f/A is minimal r-continuous. □

Remark 3.5.44:

The following diagram shows the relationship between various functions and various types of minimal and maximal r-continuous functions.

Minimal- maximal r-continuous				Maximal- Minimal r-continuous
↓				↓
Minimal r-continuous	←	Almost completely continuous	→	Maximal r-continuous
↑				↑
Minimal r-irresolute				Maximal r-irresolute

3.6 Supra r-continuous function and its properties.

Definition 3.6.1

Let (X, τ) and (Y, σ) be topological spaces and τ^* be an associated supra topology with τ(Refer section 8 of chapter 1). A function $f : (X, \tau^*) \to (Y, \sigma)$ is said to be supra r-continuous, if inverse image of each open set of Y is supra r-open in X.

Example 3.6.1

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}\}, \tau^* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}.$

Let $f : (X, \tau^*) \to (X, \tau)$ *be defined by* $f(a) = b,\ f(b) = a, f(c) = b.$ *Then* f *is supra r-continuous.*

Theorem 3.6.1

Let $(X,\ \tau)$ *and* $(Y,\ \sigma)$ *be topological spaces and* τ^* *be an associated supra topology with* τ. *Let* $f : (X, \tau^*) \to (Y, \sigma)$ *be a function. Then the following are equivalent:*

(i) f *is supra r-continuous.*

(ii) Inverse image of a closed set in Y *is supra r-closed in* X.

(iii) $Supra\ rCl(f^{-1}(A)) \subset f^{-1}(Cl(A))$ *for every* $A \subset Y$.

(iv) $f(Supra\, rCl(A)) \subset Cl(f(A))$ *for every* $A \subset X$.

(v) $f^{-1}(Int(B)) \subset Supra\ \ rInt(f^{-1}(B))$ *for every* $B \subset Y$.

Proof:

(i) $\Rightarrow$ *(ii)*

Let V be closed in Y. Then $Y - V$ is open. Since f is supra r-continuous, $f^{-1}(Y - V)$ is supra r-open. That is $f^{-1}(V)$ is supra r-closed in X.

(ii) $\Rightarrow$ *(iii)*

Let $A \subset Y$. Then $Cl(A)$ is closed in Y. By (ii), $f^{-1}(Cl(A))$ is supra r-closed.

So Supra $rCl(f^{-1}(Cl(A))) = f^{-1}(Cl(A))$.

Now $f^{-1}(A) \subset f^{-1}(Cl(A))$.

So $Supra\ rCl(f^{-1}((A))) \subset Supra\ rCl(f^{-1}(Cl(A))) = f^{-1}(Cl(A))$.

That is $Supra\ rCl(f^{-1}(A)) \subset f^{-1}(Cl(A))$.

$(iii) \Rightarrow (iv)$

Let $A \subset X$. Then $f(A) \subset Y$.

By (iii), $Supra\ rCl(f^{-1}(f(A))) \subset f^{-1}(Cl(f(A)))$.

That is Supra $rCl(A) \subset f^{-1}(Cl(f(A))$.

Hence $f(Supra\ rCl(A)) \subset Cl(f(A))$.

$(iv) \Rightarrow (v)$

Let $B \subset Y$. Then $f^{-1}(B) \subset X$.

By (iv), $f(supra\ rCl(f^{-1}(B))) \subset Cl(f(f^{-1}(B)))$ for every $f^{-1}(B) \subset X$.

That is $supra\ rCl(f^{-1}(B)) \subset f^{-1}(Cl(f(f^{-1}(B))))$.

That is $supra\ rCl(f^{-1}(B)) \subset f^{-1}(Cl(B))$.

Then $X - supra\ rCl(f^{-1}(B)) \supset X - f^{-1}(Cl(B))$.

Hence Supra $rInt(X - f^{-1}(B)) \supset f^{-1}(Int(Y - B))$.

So $X - Supra\, rInt(f^{-1}(B)) \supset X - f^{-1}(Int(B))$.

That is $f^{-1}(Int(B)) \subset Supra\ \ rInt(f^{-1}(B))$.

$(v) \Rightarrow (i)$

Let A be open in Y. Then by (v), Supra $rInt(f^{-1}(A)) \supset f^{-1}(Int(A))$.

This implies that Supra $rInt(f^{-1}(A)) \supset f^{-1}(A)$, since A is open.

But Supra $rInt(f^{-1}(A)) \subset f^{-1}(A)$.

Hence Supra $rInt(f^{-1}(A)) = f^{-1}(A)$. So $f^{-1}(A)$ is supra r-open.

So (i) holds. □

Theorem 3.6.2

Let (X, τ) and (Y, σ) be topological spaces and τ^* be associated supra topology with τ. Then

$f : (X, \tau^*) \rightarrow (Y, \sigma)$ *is supra r-continuous, if one of the following holds:*

(i) $f^{-1}(Supra\ \ rInt(B)) \subset rInt(f^{-1}(B))$ *for every* $B \subset Y$.

(ii) $rCl(f^{-1}(B)) \subset f^{-1}(Supra\ \ rCl(B))$ *for every* $B \subset Y$.

(iii) $f(rCl(A)) \subset Supra\ \ rCl(f(A))$ *for every* $A \subset X$.

Proof:

Let V be any open set of Y.

If (i) holds, $f^{-1}(Supra\ \ rInt(V)) \subset rInt(f^{-1}(V))$.

Since $Supra\, rInt(V) \subset V, f^{-1}(Supra\ \ rInt(V)) \subset f^{-1}(V) \subset rInt(f^{-1}(V))$.

But $rInt(f^{-1}(V)) \subset f^{-1}(V)$. So $f^{-1}(V)$ is regular open and so supra r-open. Hence f is supra r-continuous.

If (ii) holds, $rCl(f^{-1}(V) \subset f^{-1}(supra\ rCl(V))$ for every $V \subset Y$.

Then $rInt\, f^{-1}(Y - V) \supset f^{-1}(Supra\ rInt(Y - V))$.

Then by (i), f is supra r-continuous.

If (iii) holds, $f(rCl(f^{-1}(V)) \subset Supra\ \ rCl(V)$.

Then by (ii), f is supra r-continuous. □

Theorem 3.6.3

Every completely continuous function is supra r-continuous.

Proof:

Proof follows from the definition of completely continuous function and the result that regular open sets are supra r-open. □

Theorem 3.6.4

Every totally continuous function is supra r-continuous.

Proof:

Proof follows from the definition of totally continuous function and the result that clopen sets are regular open and regular open sets are supra r-open. □

Remark 3.6.4:

Converse of the above theorem need not be true.

Example 3.6.2

Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}\}, \tau^* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Let $f : (X, \tau^*) \to (X, \tau)$ *be defined by* $f(a) = b, f(b) = a, f(c) = c$. *Then* f *is supra r-continuous, but not totally continuous.*

Theorem 3.6.5

If X *is a discrete space, then every supra r-continuous function is totally continuous.*

Theorem 3.6.6

Every almost perfectly continuous function into a discrete space is supra r-continuous.

Theorem 3.6.7

Every almost completely continuous function into a discrete space is supra r-continuous.

Theorem 3.6.8

Let $(X, \tau), (Y, \sigma)$ *and* (Z, ν) *be topological spaces. Let* τ^* *be a supra topology associated with* τ. *If a function* $f : (X, \tau^*) \to (Y, \sigma)$ *is supra r-continuous and* $g : (Y, \sigma) \to (Z, \nu)$ *is continuous, then* $gof : (X, \tau^*) \to (Z, \nu)$ *is supra r-continuous.*

Proof:

Let $V \subset Z$ be open. Since g is continuous, $g^{-1}(V)$ is open in Y. Since f is supra r-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra r-open in X. Hence $g \circ f$ is supra r-continuous. □

Theorem 3.6.9

Composition of a supra r-continuous function and a totally continuous function is supra r-continuous.

Proof:

Let $(X, \tau), (Y, \sigma)$ and (Z, ν) be topological spaces. Let τ^* be the supra topology associated with τ. Let $f : (X, \tau^*) \to (Y, \sigma)$ be supra r-continuous, $g : (Y, \sigma) \to (Z, \nu)$ be totally continuous and $V \subset Z$ be open. Since g is totally continuous, $g^{-1}(V)$ is clopen in Y. Since f is supra r-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra r-open in X. Hence $g \circ f$ is supra r-continuous. □

Theorem 3.6.10

Composition of a supra r-continuous function and a completely continuous function is supra r-continuous.

Proof:

Let $(X, \tau), (Y, \sigma)$ and (Z, ν) be topological spaces. Let τ^* be the supra topology associated with τ. Let $f : (X, \tau^*) \to (Y, \sigma)$ be supra r-continuous, $g : (Y, \sigma) \to (Z, \nu)$ be completely continuous and $V \subset Z$ be open. Since g is completely continuous, $g^{-1}(V)$ is regular open in Y. Since f is supra r-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is supra r-open in X. Hence $g \circ f$ is supra r-continuous. □

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CHAPTER 4

Various functions on certain special spaces.

4.1 Introduction

In this chapter properties of almost perfectly continuous function and somewhat r-continuous function on certain special spaces is discussed. Discussion is also done on regular totally open function, somewhat r-open function, supra r-open function, supra r-closed function and minimal r-open function. In section 2, properties of almost perfectly continuous function and somewhat r-continuous function on certain special spaces is studied. Regular totally open function on special spaces is discussed in section 3. Section 4, is on somewhat r-open function. Supra r-open function and supra r-closed function is the topic of section 5. Minimal r-open function and their properties are introduced in section 6. Properties of graph function of various function is given in section 7.

4.2 Almost perfectly continuous and somewhat r-continuous function

Theorem 4.2.1

Let $f : X \to Y$ be an almost perfectly continuous function from a space X into a δ T_1 space Y. Then f is constant on each quasi component of X.

Proof:

Suppose f is not constant on each quasi component of X. Let a, b be two points of X that lie in the same quasi component of X such that $f(a) \neq f(b)$. Since Y is $\delta\ T_1$, there exists regular open sets U and V such that α =f(a)$\in U$ and $\beta = f(b) \in V$. Since Y is $\delta\ T_1, \{\alpha\}$ is regular closed in Y. Therefore $Y - \{\alpha\}$ is regular open. Since $f : X \to Y$ is almost perfectly continuous, $f^{-1}(Y - \{\alpha\})$ and $f^{-1}(\{\alpha\})$ are disjoint clopen sets of X. Further a $\in f^{-1}(\{\alpha\})$ and b $\in f^{-1}(Y - \{\alpha\})$, different quasi components, which is a contradiction to the assumption that b belongs to the quasi component of a. Therefore f is constant.

□

Theorem 4.2.2

If $f : X \to Y$ is a totally continuous, injective, regular open function from a clopen regular space X onto a space Y, then Y is r-regular.

Proof:

Let F be a closed set in Y and y$\notin$ F. Take $y = f(x)$. Since f is totally continuous, f^{-1}(F) is clopen in X. Let $G = f^{-1}(F)$. Then $x \notin G$. Since X is a clopen regular space, there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$. This implies $f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Since f is injective and regular open, $f(U)$ and $f(V)$ are regular open in Y and $f(U) \cap f(V) = \phi$. Thus for each closed set F and a point $y \notin F$, there exists disjoint regular open sets $f(U)$ and $f(V)$ such that $F \subset f(U)$ and $y \in f(V)$. Therefore Y is r-regular. □

Theorem 4.2.3

If $f : X \to Y$ is almost perfectly continuous, injective, regular open function from a clopen regular space X onto a space Y, then Y is ro-regular.

Proof:

Let F be a regular closed set in Y and $y \notin F$. Take $y = f(x)$. Since f is almost perfectly continuous, f^{-1}(F) is clopen in X. Let $G = f^{-1}(F)$. Then $x \notin G$. Since X is a clopen regular space, there exists disjoint open sets U and V such that $G \subset U$ and $x \in V$. This implies $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$. Since f is injective and regular open $f(U)$ and $f(V)$ are regular open in Y and $f(U) \cap f(V) = \phi$. Thus for each regular closed set F and a point $y \notin F$, there exists disjoint regular open sets $f(U)$ and $f(V)$ such that $F \subset f(U)$ and $y \in f(V)$. So Y is ro-regular. □

Theorem 4.2.4

If $f : X \to Y$ is a totally continuous, injective, regular open function from a clopen normal space X onto a space Y, then Y is r-normal.

Proof:

Let F_1, F_2 be two disjoint closed subsets of Y. Since f is totally continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are clopen in X. Since X is clopen normal, there exists open sets V_1 and V_2 such that $f^{-1}(F_1) \subset V_1$ and $f^{-1}(F_2) \subset V_2$ and $V_1 \cap V_2 = \phi$. Since f is regular open and injective, $f(V_1)$ and $f(V_2)$ are regular open and $f(V_1) \cap f(V_2) = \phi$. So Y is r-normal. □

Theorem 4.2.5

If $f : X \to Y$ is an almost perfectly continuous, injective, regular open function from a clopen normal space X onto a space Y, then Y is ro-normal.

Proof:

Let F_1, F_2 be disjoint regular closed subsets of Y. Since f is almost perfectly continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are clopen in X. Since X is clopen normal, there exists open sets V_1 and V_2 such that $f^{-1}(F_1) \subset V_1$ and $f^{-1}(F_2) \subset V_2$ and $V_1 \cap V_2 = \phi$. Since f is regular open and injective, $f(V_1)$ and $f(V_2)$ are regular open and $f(V_1) \cap f(V_2) = \phi$. So Y is ro-normal. □

Definition 4.2.1

A topological space X is said to be r-separable, if there exists a countable subset B of X which is r-dense in X.

Example 4.2.1

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{b, c\}\}$. Then $\{b, c\}$ is r-dense in X. So X is r-separable.

Theorem 4.2.6

If f is a somewhat r-continuous function from X onto Y and if X is r-separable, Y is separable.

Proof:

Let $f : X \to Y$ be somewhat r-continuous function such that X is r-separable. Then there exists a countable set B of X which is r-dense in X. Then $f(B)$ is dense in Y by theorem 3.4.1. Since B is countable and f is onto, $f(B)$ is countable. So Y is separable. □

4.3 Regular totally open function

Definition 4.3.1

A function $f : X \to Y$ is said to be regular totally open, if the image of every regular open set in X is clopen in Y.

Example 4.3.1

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{b, c\}\}, \sigma = \{Y, \phi, \{a\},\{b, c\}\}$.

Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is a regular totally open function.

Theorem 4.3.1

If a bijective function $f : X \to Y$ is regular totally open, then image of each regular closed set in X is clopen in Y.

Proof:

Let F be a regular closed set in X. Then $X - F$ is regular open in X. Since $f : X \to Y$ is regular totally open, $f(X - F)$ is clopen in Y. Since f is bijective, $f(X - F) = f(X) - f(F) = Y - f(F)$. So $Y - f(F)$ is clopen in Y. Hence $f(F)$ is clopen. □

Theorem 4.3.2

A surjective function $f : X \to Y$ is regular totally open if and only if for each subset B of Y and for each regular closed set U containing $f^{-1}(B)$, there is a clopen set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof:

Suppose $f : X \to Y$ is a surjective, regular totally open function. Let $B \subset Y$ and U be any regular closed set of X such that $f^{-1}(B) \subset U$. Then $B \subset f(U)$. Since f is regular totally open, $f(X-U)$ is clopen. So $V = Y - f(X-U)$ is a clopen subset of Y. Also it contains B and $f^{-1}(V) \subset U$.

Conversely, let F be a regular open set of X. Let $B = Y - f(F)$. Then $f^{-1}(B) = f^{-1}(Y - f(F)) \subset X - F$ and $X - F$ is regular closed. By assumption, there exists clopen set V of Y containing $B = Y - f(F)$ such that $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Now since $Y - f(F) \subset V, Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$. That is $f(F) = Y - V$ is clopen. So f is regular totally open. $\square$

Theorem 4.3.3

For any bijective function $f : X \to Y$, the following statements are equivalent.

(i.) Inverse of f is almost perfectly continuous

(ii.) f is regular totally open

Proof:

(i)$\Rightarrow$ (ii)

Suppose (i) holds. Let U be regular open in X. Since f^{-1} is almost perfectly continuous, $(f^{-1})^{-1}(U) = f(U)$ is clopen in X. so (ii) holds.

(ii)$\Rightarrow$ (i)

Suppose f is regular totally open and U is a regular open set in X. Since f is regular totally open, $f(U)$ is clopen in Y. But $f(U) = (f^{-1})^{-1}(U)$. So f^{-1} is almost perfectly continuous. $\square$

Theorem 4.3.4

The composition of two regular totally open functions is regular totally open.

Proof:

Let $f : X \to Y$ and $g : Y \to Z$ be two regular totally open functions. Consider the composition $g \circ f : X \to Z$. Let $V \subset X$ be regular open. Since f is regular totally open, $f(V)$ is clopen in Y. Since $g : Y \to Z$ is regular totally open, $g(f(V))$ is clopen in Z. Hence $g \circ f : X \to Z$ is regular totally open. □

Theorem 4.3.5

If $f : X \to Y$ is an almost perfectly continuous, regular totally open bijection from an r-normal space X to a space Y, then Y is ro-normal.

Proof:

Let A, B be two disjoint regular closed subsets of Y. Since $f : X \to Y$ is an almost perfectly continuous function, $f^{-1}(A)$ and $f^{-1}(B)$ are clopen in X. Hence they are closed. Since X is an r-normal space, there exists disjoint regular open sets U and V such that $f^{-1}(A) \subset U, f^{-1}(B) \subset V$. Then $A \subset f(U)$ and $B \subset f(V)$. Since f is a regular totally open function, $f(U)$ and $f(V)$ are clopen and so regular open. Also $f(U) \cap f(V) = f(U \cap V) = \phi$. Thus disjoint regular closed sets are separated by disjoint regular open sets. Hence Y is ro-normal. □

Theorem 4.3.6

If $f : X \to Y$ is a bijective, regular totally open function and X is clopen T_1, then Y is δT_1.

Proof:

Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is bijective, there exist distinct $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$. Since X is clopen T_1, there exists disjoint clopen sets U_{x_1} and U_{x_2} such that $x_1 \in U_{x_1}$,$x_1 \notin U_{x_2}$ and $x_2 \in U_{x_2}, x_2 \notin U_{x_1}$. Then $y_1 \in f(U_{x_1}), y_1 \notin f(U_{x_2})$ and $y_2 \in f(U_{x_2}), y_2 \notin f(U_{x_1})$ and $f(U_{x_1}) \cap f(U_{x_2}) = \phi$. Since f is regular totally open, $f(U_{x_1})$ and $f(U_{x_2})$ are clopen and hence regular open in Y. So Y is δT_1. □

Theorem 4.3.7

If $f : X \to Y$ is a bijective, regular totally open function and X is ultra Hausdorff, then Y is δT_2.

Proof:

Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since$f$ is bijective, there exist distinct $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$. Since X is ultra Hausdorff, there exists disjoint clopen and hence regular open sets U_{x_1} and U_{x_2} such that $x_1 \in U_{x_1}$, $x_2 \in U_{x_2}$. Since f is a regular totally open, injective function, $f(U_{x_1})$ and $f(U_{x_2})$ are disjoint clopen and hence regular open sets containing y_1, y_2 respectively. So Y is δT_2. □

Theorem 4.3.8

If $f : X \to Y$ is a bijective, closed, regular totally open function from an ultra regular space X, then Y is r-regular.

Proof:

Let F be a closed subset of Y with $y \notin F$. Since $f : X \to Y$ is bijective and closed, there exist x and a closed set G such that $x \notin G$ and $f(G) = F$. Since X is an ultra regular space, there exists clopen sets U_x and U_G such that $x \in U_x, G \subset U_G$ and $U_x \cap U_G = \phi$. Then $f(U_x) \cap f(U_G) = \phi$. That is $y = f(x) \in f(U_x), f(x) \notin f(U_G), F = f(G) \subset f(U_G)$. Since f is regular totally open, $f(U_x)$ and $f(U_G)$ are clopen and hence regular open in Y. So Y is r-regular. □

Theorem 4.3.9

If $f : X \to Y$ is a bijective, closed, regular totally open function from an ultra normal space X, then Y is r-normal.

Proof:

Let A, B be two disjoint closed subsets of Y. Since $f : X \to Y$ is bijective and closed, there exists disjoint closed sets G_1 and G_2 such that $f(G_1) = A$ and $f(G_2) = B$. Since X is ultra normal, there exists disjoint clopen and hence regular open sets U and V such that $G_1 \subset U, G_2 \subset V$. Since f is regular totally open, $f(U)$ and $f(V)$ are clopen and hence regular open in Y such that $f(G_1) \subset f(U), f(G_2) \subset f(V)$ with $f(U) \cap f(V) = \phi$. Hence Y is r-normal. □

Theorem 4.3.10

Let $f : X \to Y$ and $g : Y \to Z$ be two functions such that $g \circ f : X \to Z$ is regular totally open. Then the following holds.

(i.) If f is almost completely continuous and surjective, then g is regular totally open.

(ii.) If g is totally continuous and injective, then f is regular totally open.

Proof:

(i.) Let U be a regular open set in Y. Since f is almost completely continuous, $f^{-1}(U)$ is regular open in X. Since $g \circ f : X \to Z$ is regular totally open, $(g \circ f)(f^{-1}(U))$ is clopen in Z. Since f is surjective, $g(ff^{-1}(U)) = g(U)$ is clopen in Z. So $g : Y \to Z$ is regular totally open.

(ii.) Let U be a regular open set in X. Since $g \circ f$ is regular totally open, $(g \circ f)(U)$ is clopen in Z. Since g is totally continuous, $g^{-1}(g \circ f)(U)$ is clopen in Y. Since g is injective, $g^{-1}(g \circ f)(U) = f(U)$ is clopen in Y. So f is regular totally open. □

4.4 Somewhat r-open function

Definition 4.4.1

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat open [11], if for $U \in \tau$ with $U \neq \phi$, there exists an open set V in Y such that $V \neq \phi$ and $V \subset f(U)$.

Definition 4.4.2

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat r-open, if for $U \in \tau$ with $U \neq \phi$, there exists a regular open set V in Y such that $V \neq \phi$ and $V \subset f(U)$.

Example 4.4.1

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}, \sigma = \{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$.
Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. Then f is somewhat r-open.

Definition 4.4.3

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be somewhat clopen, if for $U \in \tau$ with $U \neq \phi$, there exists a clopen set V in Y such that $V \neq \phi$ and $V \subset f(U)$.

Example 4.4.2

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}, \sigma = \{Y, \phi, \{b\}, \{a, c\}\}$. *Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = b,\ f(b) = c,\ f(c) = a$. *Then* f *is somewhat clopen.*

Theorem 4.4.1

Every somewhat clopen function is somewhat r-open.

Proof:

The proof follows from the result that clopen sets are regular open . □

Remark 4.4.1:

Converse of the above theorem need not be true.

Example 4.4.3

Let $X = \{a, b, c\}, Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}\}, \sigma = \{Y, \phi, \{b\}, \{c\},\ \{b, c\}\}$.

Define a function $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = c,\ f(b) = a,\ f(c) = b$. *Then* f *is somewhat r-open, but not somewhat clopen.*

Theorem 4.4.2

If $f : X \to Y$ *is somewhat r-open, where* Y *is locally indiscrete, then* f *is somewhat clopen.*

Proof:

Let U be open in X. Since f is somewhat r-open, there exists a regular open set V in Y such that $V \subset f(U)$. But regular open sets in a locally indiscrete space are clopen. Hence V is clopen and so f is somewhat clopen. □

Theorem 4.4.3

Every somewhat r-open function is somewhat open.

Proof:

The proof follows from the result 'regular open sets are open'. □

Remark 4.4.3:

Converse of the above theorem need not be true.

Example 4.4.4

Let $X = Y = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}, \sigma = \{Y, \phi, \{c\}, \{b, c\}\}$.

Define $f : (X, \tau) \to (Y, \sigma)$ *by* $f(a) = c,\ f(b) = a,\ f(c) = b$. *Then* f *is somewhat open, but not somewhat r-open.*

Theorem 4.4.4

If $f : X \to Y$ *is somewhat open and* Y *is locally indiscrete, then* f *is somewhat r-open.*

Proof:

Let $U \in \tau$ and $U \neq \phi$. Since f is somewhat open, there exists a non empty open set V such that $V \subset f(U)$. Since Y is locally indiscrete, V is clopen and hence regular open. Hence f is somewhat r-open. □

Theorem 4.4.5

If $f : (X, \tau) \to (Y, \sigma)$ *is open and* $g : (Y, \sigma) \to (Z, \eta)$ *is somewhat r-open, then* $g \circ f : (X, \tau) \to (Z, \eta)$ *is somewhat r-open.*

Proof:

Let $U \in \tau$ and $U \neq \phi$. Since f is an open map $f(U)$ is open. Also $f(U) \neq \phi$. Since g is somewhat r- open and $f(U) \in \sigma$ with $f(U) \neq \phi$, there exists a regular open set V in η such that $V \subset g(f(U))$. So $g \circ f$ is somewhat r-open. □

Theorem 4.4.6

If $f : (X, \tau) \to (Y, \sigma)$ is a bijection, then the following are equivalent.

(i.) f is somewhat r-open.

(ii.) If C is a proper closed subset of X such that $f(C) \neq Y$, then there is a regular closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.

Proof:

(i)⇒(ii)

Let C be a proper closed subset of X such that $f(C) \neq Y$. Then $X - C$ is open in X and $X - C \neq \phi$. Since f is somewhat r-open, there exists a regular open set $V \neq \phi$ such that $V \subset f(X - C)$. Put $D = Y - V$. Clearly D is regular closed in Y. We claim that $D \neq Y$; for if $D = Y$,V=ϕ, a contradiction. Also $V \subset f(X - C)$ implies that $D = Y - V \supset Y - [f(X - C)] = f(C)$.

(ii)⇒(i)

Let U be a non empty open set in X. Put $C = X - U$. Then C is a closed subset of X and $f(C) = f(X - U) = Y - f(U)$. This implies $f(C) \neq Y$. So by (ii), there exists a regular closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$. Let $V = Y - D$. Then V is regular open and non empty. Also $V = Y - D \subset Y - f(C) = Y - (Y - f(U)) = f(U)$. So f is somewhat r-open. □

Theorem 4.4.7

Let $f : (X, \tau) \to (Y, \sigma)$ be a somewhat r-open function and A be any open subset of X. Then $f/A : (A, \tau/A) \to (Y, \sigma)$ is also somewhat r-open.

Proof:

Let $U \in \tau/A$ and $U \neq \phi$. Since U is open in A and A is open in X, U is open in X. Since $f : (X, \tau) \to (Y, \sigma)$ is somewhat r-open, there exists a non empty regular open set V in Y such that $V \subset f(U)$. Thus for any non empty open set U in τ/A, there exists a non empty regular open set V in Y such that $V \subset (f/A)(U)$. So f/A is somewhat r-open. □

Theorem 4.4.8

Let (X, τ) and (Y, σ) be any two topological spaces and $X = A \cup B$ where A and B are open subsets of X. Let $f : (X, \tau) \to (Y, \sigma)$ be a function such that f/A and f/B are somewhat r-open. Then f is also somewhat r-open.

Proof:

Let V be any open set in X such that $V \neq \phi$. Then either $(f/A)(V) \neq \phi$ or $(f/B)(V) \neq \phi$ or both.

Case (i): $(f/A)(V) \neq \phi$.

Since f/A is somewhat r-open, there exists a non empty regular open set V_1 in A such that $V_1 \subset (f/A)(V)$. Since V_1 is regular open in A and A is regular open in X, V_1 is regular open in X. So f is somewhat r-open.

Case (ii): $(f/B)(V) \neq \phi$.

This can be proved by using the same argument as in (i).

Case (iii):$(f/A)(V) \neq \phi$ and $(f/B)(V) \neq \phi$

The proof follows from the proofs of case(i) and case(ii). □

4.5 Supra r-open function and supra r-closed function

Definition 4.5.1

Let (X,τ) *and* (Y,σ) *be topological spaces and* σ^* *be supra topology associated with* σ*. The function* $f:(X,\tau)\to(Y,\sigma^*)$ *is supra r-open (resp. supra r-closed) if the image of each open (resp.closed) set in* (X,τ) *is supra r-open (resp.supra r-closed) in* (Y,σ^*)*.*

Example 4.5.1

Let $X=\{a,b,c\},\tau=\{X,\phi,\{a\}\},\tau^*=\{X,\phi,\{a\},\{b\},\{a,b\}\}$ *and* $f:(X,\tau)\to(X,\tau^*)$ *be defined by* $f(b)=a,\ f(a)=b,\ f(c)=c$*. Then* f *is supra r-open.*

Theorem 4.5.1

A map $f:(X,\tau)\to(Y,\sigma^*)$ *is supra r-open if and only if* $f(IntA)\subset Supra\ rInt(f(A))$ *for each* $A\subset X$*.*

Proof:

Suppose f is supra r-open. Then $f(IntA)$ is a supra r-open set. $f(IntA)$ is a supra r-open set contained in $f(A)$ and Supra rInt(f(A)) is the largest regular open set contained in $f(A)$ implies that $f(IntA)\subset Supra\ rInt(f(A))$, for each set $A\subset X$.

Conversely, suppose that A is an open subset of X and $f(IntA)\subset Supra\ rInt(f(A))$. Then $Int(A)=A$ and $f(A)\subset Supra\ rInt(f(A))$. Also since $Supra\ rInt(f(A))$ is the largest supra r-open set contained in $f(A)$, $Supra\ rInt(f(A))\subset f(A)$. Hence $Supra\ rInt(f(A))=f(A)$ and $f(A)$ is a supra r-open set. So f is supra r-open. □

Theorem 4.5.2

A function $f:(X,\tau)\to(Y,\sigma^*)$ *is supra r-closed if and only if Supra* $rCl(f(A))\subset$

$f(Cl(A))$ for each $A \subset X$

Proof:

Suppose f is supra r-closed. Since $f(Cl(A))$ is a supra r-closed set containing $f(A)$ and Supra $rCl(f(A))$ is the smallest supra r-closed set containing $f(A)$, Supra $rCl(f(A)) \subset f(Cl(A))$, for each $A \subset X$. Conversely suppose that A is a closed subset of X and $Supra \;\; rCl(f(A)) \subset f(Cl(A))$. Then $Cl(A) = A$ and $Supra \;\; rCl(f(A)) \subset f(A)$. Since $Supra \;\; rCl(f(A))$ is the smallest supra r-closed set containing $f(A)$, $f(A) \subset Supra \;\; rCl(f(A))$. Hence $Supra \;\; rCl(f(A)) = f(A)$. So $f(A)$ is a supra r-closed set and so f is supra r-closed.

□ □

Theorem 4.5.3

Let $(X, \tau), (Y, \sigma)$ and (Z, ν) be topological spaces. Let σ^* and ν^* be supra topologies associated with σ and ν respectively. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \nu)$. Then,

(i) if $g \circ f : (X, \tau) \to (Z, \nu^*)$ is supra r-open and $f : (X, \tau) \to (Y, \sigma)$ is a continuous surjection, then $g : (Y, \sigma) \to (Z, \nu^*)$ is supra r-open.

(ii) if $g \circ f : (X, \tau) \to (Z, \nu)$ is open and $g : (Y, \sigma) \to (Z, \nu^*)$ is supra r-continuous injection, then $f : (X, \tau) \to (Y, \sigma^*)$ is supra r-open.

Proof:

(i) Let A be an open subset of Y. Since f is a continuous surjection, $f^{-1}(A)$ is open in X. Since $g \circ f$ is supra r-open, $(g \circ f)(f^{-1}(A)) = g(A)$ is supra r-open in Z. Hence g is a supra r-open function.

(ii) Let A be an open subset of X. Since $g \circ f$ is open, $(g \circ f)(A)$ is open in Z. Since g is a supra r-continuous injection, $g^{-1}(g \circ f)(A) = f(A)$ is supra r-open in Y. Hence f is supra r-open.

□

Theorem 4.5.4

Let (X, τ) and (Y, σ) be two topological spaces and σ^* be supra topology associated with σ. Let $f : (X, \tau) \to (Y, \sigma^*)$ be a bijection. Then the following are equivalent:

(i) f is supra r-open.

(ii) f^{-1} is supra r-continuous.

Proof:

(i)⇒ (ii)

Suppose U is an open set in X. Since f is supra r- open, $f(U) = (f^{-1})^{-1}(U)$ is supra r-open in Y. So f^{-1} supra r- continuous.

(ii)⇒ (i)

Suppose (ii) holds. Let U be open in X. Since f^{-1} is supra r-continuous, $(f^{-1})^{-1}(U) = f(U)$ is supra r-open in X. so (i) holds.

□ □

Theorem 4.5.5

Let (X, τ) and (Y, σ) be two topological spaces and σ^* be supra topology associated with σ. Let $f : (X, \tau) \to (Y, \sigma^*)$ be a bijection. Then the following are equivalent:

(i) f is supra r-closed.

(ii) f^{-1} is supra r-continuous.

□

Theorem 4.5.6

If $f : (X, \tau) \to (Y, \sigma^*)$ is a bijective supra r-open map and X is T_2, then Y is $Supra\, rT_2$.

Proof:

Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since f is bijective, there exist distinct points x_1, x_2 in$X$ such that $f(x_1) = y_1, f(x_2) = y_2$. Since X is T_2, there exists disjoint open sets U_1 and U_2 such that $x_1 \in U_1$, $x_2 \in U_2$. Since f is a bijective supra r-open map, $f(U_1)$ and $f(U_2)$ are disjoint supra r- open sets containing y_1, y_2 respectively. So Y is $Supra\ rT_2$. □

4.6 Minimal r-open function

Definition 4.6.1

A function $f : (X, \tau) \to (Y, \sigma)$ is minimal r-open if the image of each regular open set in X is minimal regular open in (Y, σ).

Theorem 4.6.1

If a bijective function $f : X \to Y$ is minimal r-open, then image of each regular closed set in X is maximal regular closed in Y.

Proof:

Let F be a regular closed set in X. Then $X - F$ is regular open in X. Since $f : X \to Y$ is minimal r-open, $f(X-F)$ is minimal regular open in Y. Since f is bijective, $f(X-F) = f(X) - f(F) = Y - f(F)$, a minimal regular open set in Y. Hence $f(F)$ is maximal regular closed in Y. □

Theorem 4.6.2

Composition of two minimal r-open functions is minimal r-open.

Proof:

Let $f : X \to Y$ and $g : Y \to Z$ be minimal r-open functions. Consider the composition $g \circ f : X \to Z$. Let $V \subset X$ be regular open. Since f is a minimal r-open function, $f(V)$ is minimal regular open in Y. Since $g : Y \to Z$ is minimal r-open, $g(f(V))$ is minimal regular open in Z. Hence $g \circ f : X \to Z$ is minimal r-open. □

4.7 Graph function

4.7.1 Preliminary ideas

Definition 4.7.1

Let $f : X \to Y$ be a function. Then the graph function of f is defined by $g(x) = (x, f(x))$, for all $x \in X$.

4.8 Properties of graph function of various functions

Theorem 4.8.1

A function $f : X \to Y$ is almost perfectly continuous if its graph function is almost perfectly continuous.

Proof:

Let $g : X \to X \times Y$ be the graph function of $f : X \to Y$ and g be almost perfectly continuous. Let $V \subset Y$ be regular open in Y. Then $X \times V$ is regular open in $X \times Y$. Since g is almost perfectly continuous, g^{-1}(X × V)=$f^{-1}(V)$ is clopen in X. Therefore f is almost perfectly continuous. □

Definition 4.8.1

A subset A of the product space $X \times Y$ is supra r-closed in $X \times Y$ if for each (x, y) in $(X \times Y)$-A there exists two supra r-open sets U and V containing x and y respectively such that $(U \times V) \cap A = \phi$. A function $f : (X, \tau^*) \to (Y, \sigma^*)$ has a supra r-closed graph, if the graph $G(f) = \{(x, f(x)) : x \in X\}$ is supra r-closed in $X \times Y$.

Theorem 4.8.2

A function $f : (X, \tau^*) \to (Y, \sigma^*)$ has a supra r-closed graph if and only if for each $x \in X, y \in Y$ such that $y \neq f(x)$, there exists supra r-open sets U and V containing x and y respectively such that $f(U) \cap V = \phi$.

Proof: Suppose that $f : X \to Y$ has a supra r-closed graph. Then $G(f)$ is supra r-closed in $X \times Y$. This implies for each $(x, y) \in (X \times Y) - G(f)$, there exists two supra r-open sets U and V containing x and y respectively such that $(U \times V) \cap G(f) = \phi$. That is for each $(x, y) \notin G(f)$, $(U \times V) \cap G(f) = \phi$, where U and V are supra r-open sets containing x and y respectively. $(x, y) \notin G(f)$ implies $y \neq f(x)$ and so $f(x) \notin V$. Hence $f(U) \cap V = \phi$. Conversely, suppose that for each $x \in X, y \in Y$ such that $y \neq f(x)$, there exists supra r-open sets U and V containing x and y respectively such that $f(U) \cap V = \phi$. Then $y \notin f(U)$ and so $(x, y) \notin G(f)$. Also $(U \times V) \cap G(f) = \phi$. Hence G(f) is supra r-closed. So

$f : X \to Y$ has a supra r-closed graph. $\square$

Definition 4.8.2 *Let (X, τ) and (Y, σ) be two topological spaces and τ^* and σ^* be supra topologies associated with τ and σ respectively. Then $f : (X, \tau^*) \to (Y, \sigma^*)$ is supra* r-continuous, if inverse image of each supra r-open set is supra r-open.*

Theorem 4.8.3

If a function $(X, \tau^*) \to (Y, \sigma^*)$ is supra $^*r-continuous$ and Y is Supra rT_2, then f has a supra r-closed graph.

Proof:

Let $(x, y) \in (X \times Y)$ - $G(f)$. Then $y \neq f(x)$. Since Y is supra rT_2, there exists supra r-open sets U and V such that $f(x) \in U, y \in V$ and $U \cap V = \phi$. Since f is $supra^* r - continuous$, there exists supra r-open neighbourhood W of x such that $f(W) \subset U$. Hence $f(W) \cap V = \phi$. This implies f has a supra r-closed graph. $\square$

Definition 4.8.3

A function $f : (X, \tau^*) \to (Y, \sigma^*)$ has a strongly supra r-closed graph, if for each $(x, y) \notin G(f)$, there exists two supra r-open sets U and V containing x and y respectively such that ($(U \times Supra\ rCl(V)) \cap G(f) = \phi$.

Theorem 4.8.4

A function $f : (X, \tau^*) \to (Y, \sigma^*)$ has a strongly supra r-closed graph, if for each $(x, y) \notin G(f)$, there exists two supra r-open sets U and V containing x and y respectively such that $f(U) \cap Supra\ \ rCl(V) = \phi$.

Theorem 4.8.5

If $f : X \to Y$ be a surjective function with a strongly supra r-closed graph, then Y is a supra rT_2 space.

Proof:

Let y_1 and y_2 be two distinct points of Y. Then there exists x_1 in X such that $f(x_1) = y_1$. Then $(x_1, y_2) \notin G(f)$. Since f has a strongly supra r-closed graph, there exists two supra r-open sets U and V containing x_1 and y_2 respectively such that $f(U) \cap Supra\ rCl(V) = \phi$. Consequently $y_1 \notin V$. So Y is a Supra rT_2 space. □

Theorem 4.8.6

A function $f : X \to Y$ is somewhat r-continuous, if its graph function is somewhat r-continuous.

Proof:

Let $g : X \to X \times Y$ be the graph function of $f : X \to Y$. Suppose g is somewhat r-continuous. Let $V \subset Y$ be open in Y. Then $X \times V$ is open in $X \times Y$. Since g is somewhat r-continuous, there exists a regular open set $U \subset g^{-1}(X \times V) = f^{-1}(V)$. Therefore f is somewhat r-continuous. □

Definition 4.8.4

A subset A of the product space $X \times Y$ is somewhat r-closed in $X \times Y$ if for each $(x, y) \in (X \times Y) - A$, there exists regular open set U and an open set V containing x and y respectively such that $(U \times V) \cap A = \phi$. A function $f : X \to Y$ has a somewhat regular closed graph, if the graph $G(f) = \{(x, f(x)) : x \in X\}$ is somewhat regular closed in $X \times Y$.

Theorem 4.8.7

A function $f : X \to Y$ has a somewhat regular closed graph if and only if for each $x \in X, y \in Y$ such that $y \neq f(x)$, there exists regular open set U and an open set V containing x and y respectively such that $f(U) \cap V = \phi$.

Theorem 4.8.8

If a function $f : X \to Y$ is somewhat r-continuous and Y is T_2, f has a somewhat regular closed graph.

Proof:

Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since Y is T_2 there exists open sets U and V such that $f(x) \in U, y \in V$ and $U \cap V = \phi$. Since f is somewhat r-continuous, there exists regular open set W of x such that $f(W) \subset U$. Hence $f(W) \cap V = \phi$. This implies f has a somewhat regular closed graph. $\square$

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CONCLUSION

Through this thesis we were able to derive properties of various types of regular open sets and also to compare various types of functions and study their properties.

CHAPTER 1

Chapter 1, was on types of regular open sets. We got the following important results through the discussions of first chapter.

Properties of minimal regular open sets and maximal regular open sets

1. Intersection of a minimal regular open set and a regular open set is either empty or that minimal regular open set itself.
2. Intersection of two minimal regular open sets is either empty or both are equal.
3. Union of a maximal regular open set and a regular open set is either the whole set or that maximal regular open set itself.
4. Union of two maximal regular open sets is either the whole set or both are equal.

Properties of maximal regular closed sets and minimal regular closed sets

1. Intersection of a minimal regular closed set and a regular closed set is either empty or that minimal regular closed set itself.
2. Intersection of two minimal regular closed sets is either empty or both are equal.

3. Union of a maximal regular closed and a regular closed set is either the whole set or that maximal regular closed set itself.
4. Union of two maximal regular closed sets is either the whole set or both are equal.

Properties of weakly regular open sets

1. Union of two proper regular open sets is either weakly regular open or the whole set.
2. Intersection of two proper regular closed sets is either weakly regular closed or empty.
3. Intersection a weakly regular open set and a proper regular open set is regular open.
4. Union of a weakly regular closed set and a proper regular closed set is regular closed.
5. Union of two weakly regular open sets is either a weakly regular open set or the whole set.
6. Intersection of two weakly regular closed sets is either a weakly regular closed set or empty.
7. Union of two weakly regular closed sets is either a closed set or the whole set.
8. Intersection of two weakly regular open sets is either an open set or empty.

Properties of Supra r- open sets

1. Union of a Supra r-open set and a supra open set is a supra open set.

2. Intersection of a Supra r-open set and a supra open set need not be a supra open set.

3. Finite intersection of supra r-open sets is supra r-open.

4. Finite union of supra r-closed sets is supra r-closed.

5. Finite union of supra r-open sets may fail to be supra r-open.

6. Finite intersection of supra r-closed sets may fail to be supra r-closed.

CHAPTER 2

Separation axioms in terms of regular open sets was the topic of Chapter 2. We were able to derive properties of certain special spaces like rT_{min}, rT_{max} and rT_{weak} and some other spaces like $r-door, rT_{\frac{1}{2}}$ etc. Important results are listed below.

Hereditary and weakly hereditary properties

1. r-regularity is a hereditary property.

2. r-normality is a weakly hereditary property.

Properties of rT_{min}, rT_{max} and rT_{weak} spaces

1. rT_{min} and rT_{max} spaces will contain regular open sets of the form $A, X-A$ along with other open sets.

2. rT_{weak} spaces are of the form $\{\phi,\ X, A\}$.

3. Every pair of different minimal regular open (respectively maximal regular open) sets in rT_{min} (respectively rT_{max}) space are disjoint.
4. Union of every pair of different maximal regular open sets in an rT_{max} space is the whole space.
5. Intersection of every pair of different minimal regular open sets in an rT_{min} space is empty.
6. Every regular open subspace of an rT_{min} space is also an rT_{min} space.

Properties of spaces-rT_{max}, r-door, $rT_{\frac{1}{2}}$ etc

1. rT_{min} (respectively rT_{max}) spaces need not be δT_0 (respectively $\delta T_1,\ rT_2$) and vice-versa.
2. rT_{min} (respectively rT_{max}) space need not be $rT_{\frac{1}{2}}$ space and vice-versa.
3. rT_{min} (respectively rT_{max}) space need not be *r-door* space and vice-versa.
4. rT_{min} and rT_{max} space need not be submaximal regular space and vice-versa.

CHAPTER 3

Various functions were introduced and properties were studied in chapter 3. Comparison between the functions was also done. We got the following important results after the discussions.

Properties of almost perfectly continuous functions

1. Almost perfectly continuous functions from an r-connected space X onto any spce Y, make Y an indiscrete space.

2. If a function $f : X \to \prod Y_\lambda$ is almost perfectly continuous, then $\pi_\lambda \circ f : X \to Y_\lambda$ is almost perfectly continuous for each $\lambda \in \Lambda$,where π_λ is the projection function.

3. Restriction of an almost perfectly continuous function onto a clopen set is almost perfectly continuous.

4. Composition of an almost perfectly continuous function and an almost completely continuous function is almost perfectly continuous.

5. Composition of two almost perfectly continuous functions is almost perfectly continuous.

6. Composition of an almost perfectly continuous function and a completely continuous function is totally continuous.

7. The following diagram shows the relationship between various functions and almost perfectly continuous function.

Strongly continuous

$\downarrow$

Totally continuous $\rightarrow$ *Completely continuous*

$\downarrow$ $\qquad$ $\downarrow$

Almost perfectly continuous $\rightarrow$ Almost completely continuous

Properties of somewhat r-continuous function

1. Composition of a continuous function and a somewhat r-continuous function is somewhat r-continuous.

2. Composition of a somewhat r-continuous function and a continuous function is somewhat r-continuous.

3. If $Z = A \cap B$ and $f : Z \to Y$ is a function such that f/A and f/B are somewhat r-continuous, then f is somewhat r-continuous.

4. If X and Y are any two topological spaces, A a regular open set of X and $f : (A, \tau/A) \to (Y, \sigma)$ be somewhat r-continuous such that $f(A)$ is dense in Y, then any extension F of f is somewhat r-continuous.

5. If X and Y are topological spaces and M is an r-dense subset of X under somewhat r-continuous injective map $f : X \to Y$, then $f(M)$ is dense in Y.

6. The following diagram shows the relationship between various functions and somewhat r- continuous function.

		$Cl - super \quad continuous$		
		$\downarrow$		
Completely continuous	$\to$	Somewhat r- continuous	$\to$	Somewhat continuous
$\downarrow$		$\downarrow$		
Almost completely continuous	$\to$	$\delta - continuous$		

Properties of minimal r-continuous function and maximal r-continuous function

1. Restriction of a minimal r-continuous function on to a regular open set is a minimal r-continuous function.

2. Restriction of a maximal r-continuous function on to a regular open set is a maximal r-continuous function.

3. Composition of an almost completely continuous function and a minimal r-continuous function is minimal r- continuous.
4. Composition of an almost completely continuous function and a maximal r-continuous function is maximal r- continuous.
5. Composition of maximal r-irresolute functions is maximal r- irresolute.
6. Composition of minimal- maximal r-continuous functions need not be minimal-maximal r-continuous.
7. Composition of maximal- minimal r-continuous functions need not be maximal-minimal r-continuous.
8. Composition of a minimal r-irresolute function and a maximal-minimal r-continuous function is maximal- minimal r- continuous.
9. Composition of a minimal r-continuous function and a maximal- minimal r-continuous function is maximal r- continuous.
10. If $f : X \to Y$ and $g : Y \to Z$ are minimal-maximal r-continuous and if Y is an rT_{min} space, then $g \circ f : X \to Z$ is minimal-maximal r- continuous..
11. If $f : X \to Y$ is maximal r-irresolute and $g : Y \to Z$ is minimal- maximal r-continuous, then $g \circ f : X \to Z$ is minimal-maximal r- continuous.
12. If $f : X \to Y$ is maximal r-continuous and $g : Y \to Z$ is minimal- maximal r-continuous, then $g \circ f : X \to Z$ is minimal r- continuous.
13. If $f : X \to Y$ and $g : Y \to Z$ are maximal- minimal r-continuous and if Y is an rT_{max} space, then then $g \circ f : X \to Z$ is maximal- minimal r- continuous.

14. Restriction of a minimal r-continuous function onto a non empty regular open subset A of a topological spce X is minimal r-continuous.

15. Restriction of a maximal r-continuous function onto a non empty regular open subset A of a topological spce X is maximal r-continuous.

16. The following diagram shows the relationship between various functions and various types of minimal and maximal r-continuous functions.

Minimal- maximal r-continuous				Maximal- Minimal r-continuous
$\downarrow$				$\downarrow$
Minimal r-continuous	$\leftarrow$	Almost completely continuous	$\rightarrow$	Maximal r-continuous
$\uparrow$				$\uparrow$
Minimal r-irresolute				Maximal r-irresolute

Properties of supra r-continuous function

1. If X and Y be topological spaces, τ^* is the supra topology associated with τ and $f : (X, \tau^*) \to (Y, \sigma)$ is a function then the following are equivalent:

   (i) f is supra r-continuous.

   (ii) Inverse image of a closed set in Y is supra r-closed in X.

   (iii) Supra $rCl(f^{-1}(A)) \subset f^{-1}(Cl(A))$ for every $A \subset Y$.

   (iv) $f(Supra\, rCl(A)) \subset Cl(f(A))$ for every $A \subset X$.

   (v) $f^{-1}(Int(B)) \subset Supra\ \ rInt(f^{-1}(B))$ for every $B \subset Y$.

2. If (X, τ) and (Y, σ) are topological spaces, τ^* is the supra topology associated with τ, then $f : (X, \tau^*) \to (Y, \sigma)$ is supra r-continuous, if one of the following holds:

   (i) $f^{-1}(Supra\ \ rInt(B)) \subset rInt(f^{-1}(B))$ for every $B \subset Y$.

   (ii) $rCl(f^{-1}(B)) \subset f^{-1}(Supra\ \ rCl(B))$ for every $B \subset Y$.

   (iii) $f(rCl(A)) \subset Supra\ \ rCl(f(A))$ for every $A \subset X$.

3. Composition of a supra r-continuous function and a totally continuous function is supra r-continuous.

4. Composition of a supra r-continuous function and a completely continuous function is supra r-continuous.

CHAPTER 4

Chapter 4 , was on various functions like regular totally open, somewhat r-open etc. on certain special spaces. Through the study, following results were obtained.

Properties of almost perfectly continuous function and somewhat r-continuous function

1. Image of r-separable space under somewhat r-continuous function is separable.

2. If $f : X \to Y$ is a totally continuous, injective, regular open function from a clopen regular space X onto a space Y, then Y is r-regular.

3. If $f : X \to Y$ is an almost perfectly continuous, injective, regular open function from a clopen regular space X onto a space Y, then Y is ro-regular.

4. If $f : X \to Y$ is a totally continuous, injective, regular open function from a clopen normal space X onto a space Y, then Y is r-normal.

5. If $f : X \to Y$ is an almost perfectly continuous, injective, regular open function from a clopen normal space X onto a space Y, then Y is ro-normal.

6. If f is a somewhat r-continuous function from X onto Y and if X is r-separable, Y is separable.

Properties of regular totally open function

1. Composition of regular totally open functions is regular totally open.

2. A function $f : X \to Y$ is regular totally open if and only if $f^{-1} : Y \to X$ is almost perfectly continuous.

3. For any bijective function $f : X \to Y$ the following statements are equivalent.

   (i.) Inverse of f is almost perfectly continuous

   (ii.) f is regular totally open

4. If $f : X \to Y$ and $g : Y \to Z$ are two functions such that $g \circ f : X \to Z$ is regular totally open, then the following holds.

   (i.) If f is almost completely continuous and surjective, g is regular totally open.

   (ii.) If g is totally continuous and injective, f is regular totally open.

Properties of somewhat r-open function

1. Composition of an open map and a somewhat r-open map is somewhat r-open map.

2. Restriction of a somewhat r-open map to an open set is somewhat r-open.

3. If (X, τ) and (Y, σ) are any two topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau) \to (Y, \sigma)$ be a function such that f/A and f/B are somewhat r-open, then f is also somewhat r-open.

Properties of supra r-open function

1. A function $f : X \to Y$ is supra r-open if and only if if $f^{-1} : Y \to X$ is supra r-continuous.
2. Let $(X, \tau), (Y, \sigma)$ and (Z, ν) be topological spaces. Let σ^* and ν^* be supra topologies associated withσ and ν respectively. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \nu)$. Then,

   (i) if $g \circ f : (X, \tau) \to (Z, \nu^*)$ is supra r-open and $f : (X, \tau) \to (Y, \sigma)$ is a continuous surjection, then $g : (Y, \sigma) \to (Z, \nu^*)$ is supra r-open.

   (ii) if $g \circ f : (X, \tau) \to (Z, \nu)$ is open and $g : (Y, \sigma) \to (Z, \nu^*)$ is supra r-continuous injection, then $f : (X, \tau) \to (Y, \sigma^*)$ is supra r-open.

3. Let (X, τ) and (Y, σ) be two topological spaces. Let $f : (X, \tau) \to (Y, \sigma)$ be bijection. Then the following are equivalent:

   (i) f is supra r-open.

   (ii) f^{-1} is supra r-continuous.

Properties of minimal r-open function

1. A map $f : (X, \tau) \to (Y, \sigma)$ is minimal r-open if image of each regular open set in X is minimal regular open in (Y, σ).

2. If a bijective function $f : X \to Y$ is minimal r-open, then image of each regular closed set in X is maximal regular closed in Y.

3. Composition of minimal r-open functions is minimal r-open.

4. If $f^{-1} : (Y, \sigma) \to (X, \tau)$ is minimal r-continuous and X is rT_{min} , then $f : (X, \tau) \to (Y, \sigma)$ is minimal r-open.

Properties of graph function of various functions

1. A function $f : X \to Y$ is almost perfectly continuous if its graph function is almost perfectly continuous.

2. A function $f : X \to Y$ is somewhat r-continuous if its graph function is somewhat r-continuous.

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BIBLIOGRAPHY

[1] Abd El-Monsef M E and S N El - Deeb and R A Mahmoud: β-open sets and β-continuous mappings, Bull Fac Sci Assint Univ, 12(1), 1983, 77-90.

[2] Andrijevic D: On b-open sets, Mat.Vesnik., 48, 1996, 59-64.

[3] Arya S P and R. Gupta: On Strongly continuous mappings, Kyungpook Math.J, 14, 1974, 131-143.

[4] Balasubramanian S, C.Sandhya and M D S Saikumar: Somewhat rg-continuous and somewhat rg-open functions, Indian Journal of Engineering, 1(1), 2012.

[5] Benchalli S S, Basavaraj M Ittanagi and R S Wali: On minimal open sets and maps in topological spaces, J. Comp and Math. Sci., Vol 2(2), 2011, 208-220.

[6] Benchalli S S, Priyanka and M Bansali: Somewhat b-continuous and somewhat b-open functions in topological spaces, Int. J. Math. Anal., 4(46), 2010, 2287-2296.

[7] Benchalli S S and Umadevi I Neeli: Semitotally continuous functions in topological spaces, International Mathematical Forum, Vol 6, No. 10, 2011, 479-492.

[8] Cameron D E: Properties of s-closed spaces, Proc. Amer. Math. Soc, 72(3), 581-586.

[9] Devi R, S Sampath Kumar and M Caldas: On Supra α-open sets and $s\alpha$-continuous maps, General Mathematics, 16(2), 2008, 77-84.

[10] Ekici E: Generalisation of perfectly continuous functions, regular set connected and clopen functions, Acta. Math. Hungar, 107(3), 2005, 193-206.

[11] Gentry and Hoyle: Somewhat continuous functions, Zech. Math. J, 21(1), 1971, 5-12.

[12] Herrington L L: Properties of nearly compact spaces, Proc. Amer. Math. Soc., 45, 431-436.

[13] Jain R C: The role of regularly open sets in general topology, Ph. D Thesis, Meerut University, Institute of advanced studies, Meerut-India, 1980.

[14] Kohli J K and D Singh: Almost Cl-super continuous functions, Applied General Topology, 10(1), 2009, 1-12.

[15] Kohli J K and D Singh: Between strong continuity and almost continuity, Applied General Topology, Vol11(1), 2010, 29-42.

[16] Levin N: Semi open sets and semi continuity in topological spaces, Amer. math. monthly, 70, 1963, 36-41.

[17] Levin N: Strong continuity in topological spaces, Amer. math. monthly, 67, 1960, 269.

[18] Mashhour A S, F S Mahmoud and F H Khader: On supra topological spaces, Indian J. Pure. applied Mathematics, 14(4), 1983, 502-510.

[19] Mashhour A S,I A Hasanein and M E Abd El Monsef: Remarks on nearly compact spaces, Indian J. Pure. applied Mathematics, 12(6), 1981, 685-690.

[20] Nakaoka F and N Oda: On minimal closed sets, Proceedings of Topological spaces, theory and applications, 5, 2003, 19-21.

[21] Nakaoka F and N Oda: Some applications of minimal open sets, Int. J. Math. Sci, 27, 2001, 8471-8476.

[22] Nakaoka F and N Oda: Some properties of maximal open sets, Int. J. Math. Sci, 21, 2003, 1331-1340.

[23] Nieminen T: On ultra Pseudo compact and related spaces, Ann. Acad. Sci. Fenn. ser. AI Math., 3, 1977, 185-205.

[24] Noiri T: On δ-continuous functions, J Korean Math Soc., 16, 1980, 161-166.

[25] Reilly I L and Vamanamoorthy: On super continuous mappings, Int. J. Pure. Appl. Math. Anal.4(6), 1983, 767–772.

[26] Sekar S and J Jayakumar: On Supra I-open sets and Supra I-continuous functions, International Journal of Scientific and Engineering Research., Vol 3, Issue 5, 2012.

[27] Singal M K and Asha Rani Singal: Almost continuous mappings, Yokohoman Math. J., 16, 1968, 63-73.

[28] Singh D: Almost perfectly continuous functions, Quaest Math., 33, 2010, 1-11.

[29] Syed O R and Takashi Noiri: On Supra b-open sets and Supra b-continuity on topological spaces, European journal of pure and applied Mathematics, 3(2), 2010, 295-302.

LIST OF PUBLICATIONS

(i) Anuradha N, Baby Chacko, *Some properties of almost perfectly continuous functions in topological spaces*, International Mathematical Forum, **Vol 10, No.3**, (2015), 143-156.

(ii) Anuradha N, Baby Chacko, *Somewhat r-continuous and somewhat r-open functions*, International Journal of Pure and Applied Mathematics, **Vol 100, No.4**, (2015), 507-524.

(iii) Anuradha N, Baby Chacko, *On minimal regular open sets and maps in topological spaces*, Journal of Computer and Mathematical Sciences, **Vol 6, Issue 4, 5** & **6**, (2015), 182-192.

(ivi) Anuradha N, Baby Chacko, *On supra r-open sets and supra r-continuity* , Journal of Computer and Mathematical Sciences, **Vol 7, Issue 7, 8** & **9**, (2016), 412-419