C 82461 (Pages : 2) Name..................................

Reg. No..................................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, SEPTEMBER 2020

(CBCSS—UG)

Statistics

STA 2C 03—REGRESSION ANALYSIS AND TIME SERIES

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define correlation analysis.
2. Describe positive and negative correlation.
3. What is a Scatter diagram ?
4. Write the general expressions of regression lines x on y and y on x.
5. If the regression coefficient X on Y is 0.8, variance of X is 9 and V(Y) is 4, find the coefficient of correlation between X and Y.
6. The regression line Y on X is $2x - 3y + 5 = 0$. Identify the regression coefficient Y on X.
7. Explain, why the regression coefficients are always of same sign.
8. Define non-linear regression.
9. What are normal equations ?
10. Define time series.
11. Define secular trend.
12. Define irregular variation in time series.

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. Distinguish between qualitative and quantitative data. Give suitable examples.
14. If the ranks of 5 students in Mathematics and English are (1, 5), (2, 4), (3, 3), (4, 2) and (5, 1). Calculate the rank correlation coefficient.
15. The lines of regression X on Y and Y on X are X = 0.3125 Y + 35 and Y = 0.8X + 18. Use appropriate regression line to estimate Y when X = 4 and X when Y = 5.

Turn over

16. Explain the method of fitting of regression equation of the form $y = ax + b$ using the data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

17. Explain the method of fittin of regression equation of the form $y = ax^b$ using the data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

18. Explain the method of semi-average to measure trend in a time series.

19. Explain seasonal and cyclical variation in time series. Give example.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. Calculate Pearson's coefficient of correlation using the following data :

X	2	4	5	6	8	11
Y :	18	12	10	8	7	5

21. Using Least Square method, fit a straight line to the following time series data :—

Year (X)	2012	2013	2014	2015	2016	2017	2018
Units of sale (Y) :	125	128	133	135	140	141	143

C 82460 (Pages : 2) Name.................................

Reg. No.................................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2C 02—REGRESSION ANALYSIS AND PROBABILITY THEORY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define Karl Pearson's correlation coefficient.
2. Distinguish between Positive and Negative correlation.
3. If sum of the product of the deviation of the variables X and Y from their means is zero, find the product moment correlation coefficient.
4. State any two properties of regression coefficients.
5. If COV (X, Y) = – 30 and the regression coefficient of Y on X is – 0.3, find the variance of X.
6. Define partial and multiple correlations.
7. Define equally likely the mutually exclusive events.
8. Give classical definiton of probability.
9. State addition theorem of probability for three events.
10. If X and Y are two independent event, then P(A/B) = ————.
11. Define random variable.
12. The p.m.f., of a random variable X if $f(x) = kx, x = 1,2,3,4,5$. Find the value of k.

Section B

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. What is meant by correlation ? Explain different methods of measuring correlation.
14. Explain the difference between correlation and regression analysis.
15. Explain frequency approach to probability. What are its merits over classical approach ?

16. From the following data compute correlation between X and Y :

	X series	Y series
No. of items	15	15
Mean	25	18
Sum of squares of deviation from mean ...	136	138

and the sum of product of deviations of X and Y from their respective means is 122.

17. If $r_{12} = 0.77$, $r_{13} = 0.72$ and $r_{23} = 0.52$, find the partial correlation coefficient $r_{12.3}$ and multiple correlation coefficient $R_{1.23}$.

18. A town has two doctors A and B operating independently. If the probability that doctor A is available is 0.9 and that for B is 0.8. What is the probability that at least one doctor is available when needed ?

19. A bag contains 10 balls, two of which are red, three blue and five black. Three balls are drawn at random from the bag. What is the probability that :

    (i) Three balls are of different colours.

    (ii) Two balls are of the same colour.

Section C

Answer any **one** *question and carries* 10 *marks.*

20. A random variable X has the following probability mass function :

Values of X ...	– 2	– 1	0	1	2	3
Probability ...	0.1	k	0.2	$2k$	0.3	k

   (i) Find the value of k.

   (ii) Find $P(X > 0)$ and $P(X \leq 2)$

   (iii) Find distribution function of X.

21. Ten competitiors in a musical contest were ranked by three judges. A, B and C in the following order :

Rank by A :	1	6	5	10	3	2	4	9	7	8
Rank by B :	3	5	8	4	7	10	2	1	6	9
Rank by C :	6	4	9	8	1	2	3	10	5	7

Using rank correlation method, discuss which pair of judges has nearest approach to common taste in music.

16. From the following data compute correlation between X and Y :

	X series	Y series
No. of items	15	15
Mean	25	18
Sum of squares of deviation from mean ...	136	138

and the sum of product of deviations of X and Y from their respective means is 122.

17. If $r_{12} = 0.77$, $r_{13} = 0.72$ and $r_{23} = 0.52$, find the partial correlation coefficient $r_{12.3}$ and multiple correlation coefficient $R_{1.23}$.

18. A town has two doctors A and B operating independently. If the probability that doctor A is available is 0.9 and that for B is 0.8. What is the probability that at least one doctor is available when needed ?

19. A bag contains 10 balls, two of which are red, three blue and five black. Three balls are drawn at random from the bag. What is the probability that :

(i) Three balls are of different colours.

(ii) Two balls are of the same colour.

Section C

Answer any **one** *question and carries* 10 *marks.*

20. A random variable X has the following probability mass function :

Values of X ...	– 2	– 1	0	1	2	3
Probability ...	0.1	k	0.2	$2k$	0.3	k

(i) Find the value of k.

(ii) Find $P(X > 0)$ and $P(X \leq 2)$

(iii) Find distribution function of X.

21. Ten competitiors in a musical contest were ranked by three judges. A, B and C in the following order :

Rank by A :	1	6	5	10	3	2	4	9	7	8
Rank by B :	3	5	8	4	7	10	2	1	6	9
Rank by C :	6	4	9	8	1	2	3	10	5	7

Using rank correlation method, discuss which pair of judges has nearest approach to common taste in music.

C 82459-A (Pages : 4) Name.....................................

Reg. No.....................................

SECOND SEMESTER B.A/B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2C 02—PROBABILITY THEORY

(2019 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	Total No. of Questions : 20	Maximum : 20 Marks

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

STA 2C 02—PROBABILITY THEORY

(Multiple Choice Questions for SDE Candidates)

1. Mangoes numbered 1 through 18 are placed in a bag for delivery. Two mangoes are drawn out of the bag without replacement. Find the probability such that all the mangoes have even numbers on them ?

   (A) 43.7 %. (B) 34 %.

   (C) 6.8 %. (D) 9.3 %.

2. Two t-shirts are drawn at random in succession without replacement from a drawer containing 5 red t-shirts and 8 white t-shirts. Find the probabilities of all the possible outcomes.

   (A) 1. (B) 13.

   (C) 40. (D) 346.

3. A random variable X can take only two values, 2 and 4 i.e., P (2) = 0.45 and P (4) = 0.97. What is the Expected value of X ?

   (A) 3.8. (B) 2.9.

   (C) 4.78. (D) 5.32.

4. Three boys and four girls sit in a row with all arrangements equally likely. Let x be the probability that no two boys sit next to each other. What is x ?

   (A) $\frac{1}{7}$. (B) $\frac{2}{7}$.

   (C) $\frac{3}{7}$. (D) $\frac{4}{7}$.

5. If X is A discrete random variable and $f(x)$ is the probability of X, then the expected value of this random variable is equal to :

   (A) $\sum f(x)$. (B) $\sum [x + f(x)]$.

   (C) $\sum f(x) + x$. (D) $\sum x f(x)$.

6. Which of the following is not possible in probability distribution ?

   (A) $p(x) \geq 0$. (B) $\sum p(x) = 1$.

   (C) $\sum xp(x) = 2$. (D) $p(x) = -0.5$.

7. A discrete probability distribution may be represented by :

   (A) Table. (B) Graph.

   (C) Mathematical equation. (D) All of the above.

8. A fair die is rolled. Probability of getting even face or face more than 4 is :

(A) 1/3.

(B) 2/3.

(C) 1/2.

(D) 5/6.

9. If A and B are two not-independent events, then the probability that both A and B will happen together is :

(A) $P(A \cup B) = P(A)\,P(B/A)$.

(B) $P(A \cup B) = P(A)\,P(B)$.

(C) $P(A \cup B) = P(A) + P(B)$.

(D) $P(A \cup B) = P(A)$.

10. A random variable is said to be ———————— if its range set is either finite or countably infinite.

(A) Continuous.

(B) Discrete.

(C) Both (A) and (B).

(D) None of these.

11. A continuous random variable follows a standard normal distribution if its probability distribution function is given by :

(A) $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{z^2}{2}}, -\infty < z < \infty$.

(B) $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty$.

(C) $f(x) = \frac{1}{2\pi} e^{\frac{z^2}{2}}, -\infty < z < \infty$.

(D) $f(x) = \frac{1}{2\pi} e^{-\frac{z^2}{2}}, -\infty < z < \infty$.

12. If we have observations with respective frequencies $f_1, f_2, \ldots\ldots, f_n$, then its arithmetic mean is given by :

(A) $\frac{1}{N} \sum_{i=1}^{n} f_i \left(X_i - \bar{X}\right)^2$.

(B) $\sum_{i=1}^{n} f_i X_i$.

(C) $\frac{1}{N} \sum_{i=1}^{n} f_i X_i$.

(D) None of these.

13. Let X be a random variable with the following probability distribution. Find $E\left(X^2\right)$:

x :	-3	6	9
$P(X = x)$:	1/6	1/2	1/3

(A) 11/2.

(B) 93/2.

(C) 44/2.

(D) 209.

14. If X and Y are two random variables, then $E(X+Y)$ is :

(A) $E(X) \cdot E(Y)$.

(B) $E(X) + E(Y)$.

(C) $E(X) + E(Y) - E(X, Y)$.

(D) $E(X) + E(Y) - E(X) \cdot E(Y)$.

15. If X and Y are independent random variables, which of the following is incorrect ?

(A) $V(X+Y) = V(X) + V(Y)$.

(B) $V(X-Y) = V(X) + V(Y)$.

(C) $\rho_{X,Y} \neq 0$.

(D) $\text{cov}(X, Y) = 0$.

16. A curve is said to be leptokurtic if :

(A) $\mu_4/\mu_2^2 - 3 > 0$.

(B) $\mu_4/\mu_2^2 - 3 < 0$.

(C) $\mu_4/\mu_2^2 - 3 = 0$.

(D) None of these.

17. Which of the following is not the variance of X ?

(A) $E\left[X - E\left(X^2\right)\right]$.

(B) $E\left(X^2\right) - \left[E(X)\right]^2$.

(C) $E\left[X - E(X)\right]^2$.

(D) None of these.

18. $V(aX + b) =$ ——————.

(A) $a^2 \cdot V(X)$.

(B) $a^2 \cdot V(X) + b^2$.

(C) $a \cdot V(X) + b$.

(D) $a \cdot V(X)$.

19. Let X be the number of heads obtained in four tosses of a fair coin. Find E (X) :

(A) 16/16.

(B) 32/16.

(C) 33/16.

(D) None of these.

20. Given below is the probability distribution of X. What is $E(X)$?

X	:	– 2	– 1	0	1	2
P (X)	:	0.1	0.2	0.3	k	$3k$

(A) 0.

(B) 0.3.

(C) – 0.1.

(D) None of these.

C 82459 (Pages : 2) Name..................................

Reg. No...............................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2C 02—PROBABILITY THEORY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. Define sample space, event of a random experiment.
2. Explain mutually exclusive and exhaustive events.
3. If P(A) = 0.6, P (A ∪ B) = 0.8, find P(B) when A and B are independent.
4. Define P(A/B), where A and B are two events. Also state the multiplication theorem on probability.
5. Differentiate discrete and continuous random variables.
6. Find k, if $f(x) = kx^2$, for $0 < x < 1$ is a probability density function of X.
7. For a random variable X with possible values 1, 2 and 3, identify with reason, the values F(0.5) and F(3.2) where F is the distribution function of X.
8. Define Mathematical expectation of a discrete random variable X. Also show that, for a random variable X, $[E(X)]^2 \leq E(X^2)$ if the expectations exist.
9. If $M_X(t)$ is the m.g.f. of X, identify the m.g.f. of 2X – 5.
10. Find the characteristic function of X, where P(X = x) = 0.5; for x = 0.1.
11. Express coefficient of correlation between two random variables X and Y in terms of expectations.
12. If the joint p.d.f. of X and Y is $f(x, y) = 1$, for $0 < x < 1; 0 < y < 1$, find P(X > 0.2/Y > 0.6).

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. For two events A and B, P(A) = 0.4, P(B) = 0.3, P(A ∩ B) = 0.2. Find (i) P(At least one of A and B to happen) ; (ii) P(Exactly one of A and B to happen).
14. For two events A and B, prove that $P(A \cup B)/C = P(A/C) + P(B/C) - P(A \cap B/C)$.

Turn over

15. Identify the distribution function of X and sketch its graph when the possible values of X are – 1, 0, 1 and 2 with respective probabilities 0.2, 0.35, 0.4 and 0.05.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -\log_e X$.

17. Given the p.d.f. of X as $f(x) = e^{-x}$, for $0 < x < \infty$. Find the m.g.f. of X and hence the variance of X using m.g.f.

18. The first three raw moments of X are $\lambda, \lambda^2 + \lambda$ and $\lambda^3 + 3\lambda^2 + \lambda$. Obtain the coefficient of skewness of X and identify the condition for symmetry.

19. State and prove Cauchy-Schwartz inequality for two random variables X and Y.

Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. State and prove Bayes' theorem. Result of a survey from a college consists of 40 % boys and 60% girls on a recently released film, reveals that 35 % of the boys like the film but 30 % of the girls not like the film. A randomly selected student from this college likes the film. What is the probability that the student is a girl ?

21. (a) State and prove the multiplication theorem on expectation for the two random variables X and Y.

    (b) If the joint p.d.f. of (X, Y) is $f(x, y) = cxy$, for $0 < x < y < 1$.

    (i) Find the value of c ; (ii) Verify whether X and Y are independent.

C 82458

(Pages : 2)

Name.....................................

Reg. No.................................

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Statistics

STA 2B 02—BIVARIATE RANDOM VARIABLE AND PROBABILITY DISTRIBUTIONS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 25.

1. Define Mathematical expectation. If $X \geq 0$, show that $E(X) \geq 0$.
2. For any random variable X, where E(X) exists, show that the first central moment is zero.
3. Define moment generating function of a random variable X.
4. Obtain the characteristic function of a Bernoulli random variable.
5. Define joint probability mass function of two discrete random variables X and Y.
6. Obtain the joint distribution function of X and Y where the joint p.d.f. is given by $f(x, y) = 1, 0 < x < 1, 0 < y < 1$.
7. Find c, if $f(x, y) = c(x + y), x = 0, 1; y = 1, 2$ is a joint p.m.f. of (X, Y).
8. For a random variable X, if $V(X) = 4$, find $V(X - 4)$.
9. For two random variables X and Y show that $Cov(aX, bY) = ab\,Cov(X, Y)$, a and b are constants.
10. Define Binomial distribution.
11. Obtain the mean of a random variable denoting the number of failures before the first success in a random experiment with only two outcomes success and failure with a constant probability of success p in each trial.
12. If X follows discrete uniform distribution over $[1, 2, 3, \ldots n]$, find E(X).
13. If the mean of a random variable X following Poisson distribution is 5, find $P(X > 0)$.
14. Define hyper geometric distribution.
15. State Weak Law of Large Numbers.

Turn over

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 *marks.*

Maximum marks that can be scored from this part is 35.

16. Prove that $E(aX+b) = aE(X)+b$, for a random variable X and for the constants a and b. Hence deduce that the Mathematical expectation of a constant is the constant itself.

17. State and prove Cauchy-Schwartz inequality.

18. The p.d.f. of X is $f(x) = e^{-x}$ for $x > 0$. Obtain the m.g.f., and first raw moment of X.

19. If $f(x, y) = \frac{x+y}{18}, x, y = 0,1,2$ is a joint p.m.f. of (X, Y). Find E (X/Y = 1).

20. Find the m.g.f. of X following Poisson distribution with parameter λ and hence state and prove the additive property of Poisson distribution.

21. One man decided to continue a game until his 6th success. The probability of success in any trial of the game is 0.4. Calculate the probability that he will have to play 10 trials.

22. Prove that the conditional distribution of X given Y = y of two independent random variables X and Y following Poisson distributions is binomial distribution.

23. State and prove Chebycheve's inequality.

Part C (Essay Type Questions)

Answer any **two** *questions.*

Each question carries 10 *marks.*

Maximum marks that can be scored from this part is 20.

24. State and prove the addition and multiplication theorems on expectation for two random variables X and Y.

25. Given the joint p.d.f. of two random variables X and Y as, $f(x,y) = \begin{cases} k, & \text{for } 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$ Find

    (i) k ; (ii) coefficient of correlation between X and Y.

26. If $\mu_{r-1}, \mu_r + \mu_{r+1}$ respectively be the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ central moment of a random variable X following binomial distribution with parameters n and p show that $\mu_{r+1} = pq\left[\frac{d}{dp}\mu_r + nr\mu_{r-1}\right]$.

    Obtain the coefficient of skewness β_1 and prove that the distribution becomes symmetric when $p = 0.5$.

27. Find the mean and variance of a random variable X following geometric distribution with parameter p. Also state and prove the lack of memory property of this distribution.