D 32742 (Pages : 2) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2022

Statistics

MST1C01—ANALYTICAL TOOLS FOR STATISTICS—I

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. Define continuity of a complex function.
2. Find the Laplace transform of $\sin(at)$.
3. What do you mean by a limit point in a complex plane ?
4. What are meant by stationary points in optimization ?
5. What is essential singularity ?
6. State Taylor's theorem for real functions of two variables.
7. Examine for maxima or minima for the function $f(x,y) = x^3y^2(1-x-y)$.

(4 × 2 = 8)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. State and prove Fourier integral theorem.
9. Let $f(x,y) = \begin{cases} \dfrac{xy\left(x^2-y^2\right)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$ Check whether the mixed partial derivatives are equal or not at the origin.
10. Explain the concept of Fourier transform.

Turn over

11. Give convolution property. Using this property find the inverse Laplace transform of $\frac{1}{s\left(s^2+a^2\right)}$.

12. If $xyz = abc$, where a, b and c are constants, find the minimum value of $bcx + cay + abz$.

13. For the function $f(z) = (z-1)^{-2}$, show that $z = 1$ is a pole. Also determine its kind ?

14. Find the minimum value of $xy + \frac{1}{x} + \frac{1}{y}$.

(4 × 3 = 12)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Evaluate the integral $\int_C \cosh\left(\frac{1}{z^2}\right) dz$, where C is positiely oriented unit circle $|z| = 1$.

16. Solve $y'' + 9y = \cos(2t)$ if $y(0) = 1$, $y(\pi/2) = -1$.

17. State and prove a set of necessary and sufficient conditions for a complex function to be analytic.

18. State and prove Cauchy's residue theorem.

(2 × 5 = 10)

D 32743 (Pages : 2) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2022

Statistics

MST 1C 02—ANALYTICAL TOOLS FOR STATISTICS—II

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. Define linear dependence and independence.

2. Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ a_1^2 & a_2^2 & a_3^2 \end{vmatrix}$.

3. Find the rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 6 & 9 \end{bmatrix}$.

4. Show that the characteristics roots of an orthogonal matrix are ± 1.

5. Define geometric multiplicity.

6. When do you say that a square matrix is positive definite ?

7. Define Moore-Penrose g-inverse of a matrix.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Find the dimension of the vector space spanned by (1, 1, 1, 1), (1, 2, 3, 2), (2, 5, 6, 4) and (2, 6, 8, 5).

9. Show that the mapping $T : R^2 \to R^2$ defined by $T(x, y) = (x + y, x)$ is a linear transformation.

Turn over

10. For any two matrices A and B with rank $\rho(.)$, show that $\rho(A+B) \leq \rho(A) + \rho(B)$.

11. Show that the characteristic roots of an idempotent matrix are 0 or 1.

12. Show that the geometric multiplicity cannot exced the algebraic multiplicity.

13. If A^+ is the Moore-Penrose g-inverse of a matrix A. Then show that $(A^+)^+ = A$.

14. Investigate the nature of the quadratic form $x^2 + 6xy - y^2 - 2yz + z^2$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Show that the vectors (1, 1, 1, 1), (1, 2, 3), (1, 5, 8) span R^3.

16. State and prove the rank-nullity theorem.

17. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Also find A^{-2}.

18. Find the g-inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

(2 × 5 = 10 weightage)

D 32744 (Pages : 2) Name.............................

Reg. No.............................

FIRST SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2022

Statistics

MST 1C 03—DISTRIBUTION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. Let X andY be independent negative binomial random variables with parameters (r_1, p) and (r_2, p) respectively. Obtain the conditional distribution of $X|(X + Y)$.
2. Obtain the probability generating function of binomial distribution. Hence obtain the mean.
3. If X and Y be independent $N(0, \sigma^2)$ random variables, then derive the PDF of $\frac{X}{Y}$.
4. Let $X_1, X_2, \ldots, X_n$ be iid random variables having beta distribution of first kind with parameters $(\alpha, 1)$. Obtain the distribution of max $(X_1, X_2, \ldots, X_n)$.
5. Let X and Y be iid radnom variables with PMF $p(x) = \frac{1}{2}, x = 1, 2.$ If Z = XY, then show that X and Z are independent.
6. Derive the PDF of rth order statistic of a random sample size n taken from a continuous population.
7. Establish the additive property of chi square random variables.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Establish the recurrence relation for factorial moments of generalized power series distribution.
9. Let X be a non-negative integer valued random variable. Prove that X has lack of memory property if and only if it is geometric random variable.

Turn over

10. Define Laplace distribution. Derive its moment generating function and hence find the mean and variance.

11. Let X and Y be iid N (0, 1) random variabels. Show that X + Y and X – Y are independent.

12. If X and Y have the joint PDF $f(x, y) = 8xy, 0 < x < y < 1$ and $f(x, y) = 0$, otherwise. Obtain $E(Y | X = x)$ and $V(Y | X = x)$.

13. If X follows F distribution with (m, n) degrees of freedom, then show that $Y = \frac{1}{1 + \frac{m}{n} X}$ follows beta distribution of first kind with parameters $\left(\frac{n}{2}, \frac{m}{2}\right)$.

14. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that the sample mean $\bar{X}$ and the sample variance S^2 are independently distributed.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

15. If the bivariate random vector (X, Y) follows trinomial distribution with parameters (n, p_1, p_2), then derive the MGF of (X, Y), and hence find the marginal distributions. Also show that cov (X, Y) = $-np_1p_2$.

16. Define Pearson system of distributions. Derive Gamma and Beta distributions as a special case of Pearson system.

17. Let $X_{1:n}, X_{2:n}, \ldots X_{n:n}$ be the order statistics of a random sample of size n taken from the PDF $f(x) = 1, 0 < x < 1$ and $f(x) = 0$, otherwise. If $Y_i = \frac{X_{i:n}}{X_{i+1:n}}$, $i = 1, 2, \ldots, n-1$, and $Y_n = X_{n:n}$, then show that Y_i's are independent. Find the PDF of Y_i, $i = 1, 2, \ldots, n$.

18. Define non-central t-distribution. Derive its PDF.

(2 × 5 = 10 weightage)

D 32745 (Pages : 2) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2022

Statistics

MST 1C 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. Define limit of a sequence of sets. Find the limit of a monotone increasing and monotone decreasing sequence of sets.
2. Explain general probability space and induced probability space.
3. State and prove correspondence theorem.
4. Define expectation of a random variable. Give an example of a random variable whose expectation doesn't exist.
5. State Levy continuity theorem. Give any one application of continuity theorem.
6. If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$, then show that $aX_n \xrightarrow{P} aX$ where a is a real number and $\xrightarrow{P}$ means convergence in probability.
7. State Kolmogrov three series theorem.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 weightage.

8. Define field with an example. Also show that a field is closed under finite union.
9. (a) Establish the properties of a distribution function.

   (b) Show that the set of discontinuity points of a distribution function $F_X(x)$ is atmost countable.
10. If X and Y be two random variables defined on a probability space $(\Omega, \mathcal{A}, P)$, then show that $E|XY| \leq E^{\frac{1}{r}} |X|^r E^{\frac{1}{s}} |X|^s$, where $r > 1$ with $r^{-1} + s^{-1} = 1$ and $E^{\frac{1}{P}} |X|^r = E\left[\sqrt[P]{|X|^r}\right]$.

Turn over

11. State and prove Helly-Bray theorem.
12. Establish the inter-relations between convergence in probability and convergence in rth mean.
13. State and prove the weak law of large numbers in the case of independent and identically distributed random variables.
14. Show that Liapounov conditions of central limit theorem implies Lindberg conditions of central limit theorem.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. (a) Show that a necessary and sufficient condition for a given function is measurable is that its positive and negative parts are measurable.

    (b) Obtain the positive and negative part of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax^2 + bx + c$.
16. State and prove Kolmogorov zero one law.
17. (a) Define convergence in distribution and almost sure convergence of a sequence of random variables.

    (b) Prove or disprove convergence in distribution implies convergence in probability.
18. State and prove classical central limit theorem. Also specify any two applications of central limit theorem.

(2 × 5 = 10 weightage)