D 11718 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 24—APPLIED ALGORITHM AND ANALYSIS OF MULTITYPE AND BIG DATA

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. Identify the significance gibbs sampling of EM algorithm.
2. Enlist the strengths of a perceptual mapping in big data.
3. Mathematically represent the linear support vector classifier with an equation.
4. Why Metadata is added to the data during the processing ?
5. List the purpose of data acquisition and filtering stage.
6. Identify the significance of data augmentation in EM algorithm.
7. Define a hyper plane.

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Compare the purpose of expectation step and maximization step in EM algorithm.
9. Distinguish Aggregate versus Disaggregate Analysis.
10. How business intelligence can improve the performance of an organization ?
11. Illustrate the procedure of one *vs* all classification in SVM.

12. What is the advantage of using a kernel rather than simply enlarging the feature space using functions of the original features in SVM ?

13. How do big data analytics differ from traditional data analysis ?

14. Compare metric *vs* non metric methods.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Describe EM as a maximization-maximization procedure.

16. Elaborate on Maximal - Margin classifier.

17. Explain the characteristics of big data.

18. Discuss Decision framework for perpetual mapping.

(2 × 5 = 10 weightage)

D 11717 (Pages : 2) Name.....................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 22—NON PARAMETRIC STATISTICAL METHODS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage each.

1. Distinguish between nominal and ordinal scales of measurements with examples.
2. Why do we use QQ plot ? How it overcomes the limitations of histogram ?
3. Define cross-validation estimator of risk
4. Describe how Lilliefors's test is different from Kolmogrov-Smirnov test for the problem of goodness of fit test.
5. Define Crammer- Von mises test statistic for identical population.
6. How to find confidence interval using the kernel density estimator ?
7. What is the use of Jarque- Bera test ? Define Jarque-Bera test statistic.

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage each.*

8. Explain (i) Crammers contingency co-efficient ; and (ii) Pearson contingency co-efficient for a two way contingency table.

9. Stating the assumptions, describe the McNemar test procedure.

10. Compare Anderson Darling and Shapiro -Wilk tests for normality.

11. Define empirical distribution function. Show that it is an unbiased and consistent estimator of population distribution function.

12. Explain bias variance trade off and the problem of choosing smoothing parameters in nonparametric inference.

13. Describe bootstrap method for estimating variance of an estimator.

14. Describe Friedman's test.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage each.*

15. (i) Define Run. What is run test for randomness ? Explain Wald-Wolfowitz run test for two sample problem

 (ii) Describe Lilliefors's test for exponential distribution

16. (i) Define Phi coefficient for a 2 × 2 table. How to interpret Phi co-efficient ?

 (ii) What is the difference between ratio scale and interval scale of measurements? Give examples

 (iii) Define Kaplan Meier estimator. What are its assumptions ? How to calculate the Kaplan Meier survival curve ?

17. (i) Explain Kruskal-Wallis test for one way analysis of variance.

 (ii) Define Quantiles. Write short notes on the hypothesis testing for a population quantile.

18. (i) Explain histogram density and kernel density estimators. Find the bias of histogram density estimator.

 (ii) What is a rug plot used in a density plot ? How do you make a drug plot in R ?

(2 × 5 = 10 weightage)

D 11716 **(Pages : 2)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 19—STATISTICAL MACHINE LEARNING—I

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer any **four** *questions.*
Each question carries 2 *weightage.*

1. List the features of supervised learning with an example.
2. Define Overfitting.
3. Identify the scenario when cross validation is used for evaluation.
4. Compare model assessment and model selection.
5. Give the equation for computing least squared error.
6. Write the significance of kernel density estimation.
7. How cubic spline can be represented ?

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Outline the concept of the Bias-Variance decomposition in machine learning.
9. Explain the curse of dimensionality problem.
10. Outline the optimism of the training error rate.
11. Define Vapnik-Chervonenkis Dimension. List its significance.
12. Describe Statistical method for Joint Distributions Pr (X, Y).

Turn over

13. Discuss Bumping method.

14. Compare bootstrap and bagging model.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Explain any *two* linear methods of regression.

16. Describe Naive Bayes Classifier.

17. Elaborate on Wavelet Smoothing techniques.

18. Illustrate EM algorithm with an example.

(2 × 5 = 10 weightage)

D 11715 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 13—BIOSTATISTICS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries weightage of 2.

1. What are the different types of biological data ?
2. What do you mean by hazard function ?
3. What is progressive censoring ? How is it useful ?
4. What is hazard ratio ?
5. Define a stochastic epidemic model.
6. What is Mendel's law of segregation ?
7. What are the uses of permutation tests ?

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each question carries weightage of 3.

8. List out any 2 the survival distributions having bathtub shape hazard function.
9. Describe Cox's F-test with applications.

Turn over

10. Find the Kaplan Meier estimate for the following data : 3, 5, 5+, 6, 7, 8, 9, 10, 10+. (Here + denotes the censored life time) and briefly explain the method also.

11. Discuss Actuarial Estimation Method.

12. Write a short note on stochastic epidemic models.

13. Describe Assortative mating and Disassortative mating.

14. List out the ethics in Randomized Clinical Trials.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

15. Explain the Survival Distributions : (i) Exponential; (ii) Gamma; (iii) Log-normal.

16. Estimate the mean survival time and variance for type I and type II censored data.

17. Explain the significance of logistic regression.

18. Describe the procedure of detection and estimation of linkage in heredity.

(2 × 5 = 10 weightage)

D 11714 (Pages : 2) Name..

Reg. No......................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 10—STATISTICAL QUALITY CONTROL

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. How will you estimate the process standard deviation from the mean range of samples of constantsize drawn from a continuous production process ?
2. Obtain the OC function of p-chart.
3. Define the terms : Producers risk and AQL.
4. In what way military standard tables different from Dodge Romig Tables.
5. What do you mean by ARL of a control chart ?
6. What do you mean by C_p index and C_{pk}index ?
7. Illustrate the concept of shift in the process average. What type of control charts is useful ? In identifying shifts of small magnitude in process average ?

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. What is meant by curtailed inspection ? Show that a single sampling plan can be made more efficient by curtailment.

9. Describe the use of sequential sampling plans in industry.

10. Obtain OC and ASN functions for the following plan :

    If the number of defectives in the first sample of size 6 is zero, accept the lot. If it exceeds3, reject the lot, otherwise take a second sample of size 6. If the number of defectives in the combined sample is4 or less, accept the lot, otherwise reject the lot.

11. Discuss the statistical basis of control chart technique.

12. Explain how you can uses-chart in the place of R- chart. Indicate the merits and demerits of the s-chart.

13. Explain how a process can be under control from statistical point of view but still be economically unsatisfactory. State the precautions to be taken to guard against such situation.

14. Defects on an automobile can be classified as A-very serious, B-serious, C-moderately serious, D-minor. Describe an appropriate control chart for controlling number of defects per unit.

($4 \times 3 = 12$ weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Describe rectification inspection plans. What is AOQ function of a sampling plan ? Derive the AOQ function of a single sampling plan.

16. Find ASN function of a double sampling plan. Bring out the difference between ASN function and ATI function of a double sampling plan.

17. Explain the construction and use of group control charts.

18. Describe in detail the role of statistical process control in statistical quality control.

($2 \times 5 = 10$ weightage)

D 11712 (Pages : 2) Name..............................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3E 02—TIME SERIES ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries a weightage of 2.

1. Explain forecasting procedure based on exponential smoothing.
2. Define weak stationarity. Examine whether an i.i.d. sequence of random variables is weak stationary.
3. Define autocorrelation function and partial autocorrelation function.
4. Define spectral distribution of a weakly stationary stochastic process. State its properties.
5. For a stationary AR (1) process obtain l-step ahead forecast.
6. If the process $Z_t = \phi_0 + \phi_1 Z_{t-1} + \phi Z_{t-2} + \varepsilon_t$ is stationary, obtain its mean. Write the expression for ACF in terms of $\gamma_k = \text{cov}(Z_t, Z_{t-k})$.
7. Obtain the variance of a linear process $Z_t = \sum_{t=0}^{\infty} \psi_i \varepsilon_{t-i}$.

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries a weightage of 3.

8. Explain how will you test for trend and seasonality present in a time series.

9. Give example of two MA processes having the same ACF.

10. Consider the process $Z_t = \varepsilon_t \varepsilon_{t-1} \varepsilon \sim N\left(0, \sigma^2\right)$. Find mean and autocovariance function. Is the process stationary ?

11. Describe the Box-Jenkins approach of analysis of a time series data.

12. Derive the spectral density of ARMA (p, q) process given by $\Phi(B)Z_t = \Theta(B)\varepsilon_t$.

13. Obtain the stationarity condition for AR (2) process.

14. Show that the function $r(h) = \begin{cases} 1 & \text{if } h = 0 \\ \rho & \text{if } h = \pm 1 \\ 0 & \text{Otherwise} \end{cases}$ is an autocovariance function if and only if $|\rho| < 1/2$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weight of 5.

15. Explain important components of a time series. Describe Winter seasonal forecast procedure for additive and multiplicative models.

16. Obtain invertibility condition for moving average process of order p. Deduce the same for MA (2) process.

17. Derive Yule-Walker equation for estimation of parameters of AR process. Obtain the expression for PACF of lag k of an MA (1) process.

18. Describe tests for stationarity of the estimated noise sequence in a time series data.

(2 × 5 = 10 weightage)

D 11710 (Pages : 2) Name..

Reg. No..

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3C 10—STOCHASTIC PROCESS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. How does a stochastic process differ from an ordinary random variable ?
2. Define a reducible Markov chain. Give the transition probability matrix of a reducible Markov chain.
3. Define a transient Markov chain. Give an example of a transient Markov chain.
4. Give a practical situation that can be described using a compound Poisson process.
5. Define infinitesimal generators. Give any *one* relation of infinitesimal generator matrix with the transition probability matrix of the corresponding process.
6. State key renewal theorem.
7. Comment on the nature of stationarity of Gaussian processes.

($4 \times 2 = 8$ weightage)

Turn over

Section B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. What do you mean by a Markov process ? Give an example of a Markov process and establish its Markov property.
9. Explain Galton-Watson branching process in the context of COVID-19.
10. Establish the relation of Poisson process with geometric and negative binomial distributions.
11. State and prove elementary renewal theorem.
12. Explain pure birth and death processes. Obtain the forward differential equation of it.
13. Prove that, with probability one, the set of local maxima of the Brownian path W(t) is countable.
14. Distinguish between weakly and strongly stationary processes with the help of suitable examples.

(4 × 3 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Obtain the expression of the expected duration of the game in a Gambler's ruin problem.
16. Stating clearly the postulates, derive the probability distribution of the Poisson process.
17. State and prove central limit theorem of renewals.
18. Carryout the stationary equilibrium analysis of M/G/1 queues.

(2 × 5 = 10 weightage)

D 11709 (Pages : 2) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 3C 09—APPLIED REGRESSION ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries a weightage of 2.

1. What is the use of indicator variables in regression modeling ?
2. What is meant by heteroscedasticity ?
3. Obtain an unbiased estimator for the parameter symbol of the regression model $y = ax + b + \varepsilon$.
4. What are the advantages of adjusted R^2 over co-efficient of determination ?
5. What are the assumptions of general linear regression model ?
6. Discuss logistic regression model.
7. What are kernel smoothers in non-parametric regression ?

(4 × 2 = 8 weightage)

Turn over

Part B

Answer any **four** *questions.*
Each question carries a weightage of 3.

8. Discuss the method of splines in the context of polynomial regression.
9. Define multicollinearity. What are the consequences of multicollinearity in linear regression ?
10. What is polynomial regression model ? Explain a method for its estimation.
11. State and prove Gauss Markov theorem.
12. Write a note on stepwise regression methods.
13. Derive the estimate of the parameters of a linear regression model using the method of least squares. Under the assumption of normality show that mle coincides with least square estimates.
14. Derive the procedure for testing the hypothesis that all of the regression slopes are zero.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

15. For the linear regression model $Y = X\beta + \varepsilon, \varepsilon \sim N\left(0, \sigma^2 I\right)$ find the distribution of residual sum of squares. Show that mle of β and $e'e$ are independent, where e is the residual.
16. Explain Poisson Regression. Explain method of estimation of parameters in Poisson regression.
17. Discuss autocorrelation. What are the consequences of autocorrelation ? Discuss how to detect autocorrelation in multiple linear regression.
18. Discuss 'Generalized Least Squares' and obtain the form of the GLS estimate.

(2 × 5 = 10 weightage)

D 11388 (Pages : 3) Name.......................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (SUPPLEMENTARY) EXAMINATION NOVEMBER 2021

(CUCSS)

Statistics

STA 3C 11—STOCHASTIC PROCESSES

(2010 to 2018 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. What is meant by a covariance stationary process ?
2. Give an example of a Markov chain.
3. Define first passage time distribution.
4. State ergodic theorem.
5. What are the postulates of a Poisson process ?
6. Mention any one application of compound Poisson process.
7. What is a time dependent Poisson process ?
8. Define Brownian motion process.
9. What is Ornstein -Uhlenbeck process ?
10. Describe Polya's urn model.
11. Define Galton-Watson branching process.
12. State the relation between the probability of ultimate extinction and mean of the offspring distribution.

($12 \times 1 = 12$ weightage)

Part B

Answer any **eight** *questions.*
Each question carries 2 *weightage.*

13. State and prove Chapman-Kolmogorov equation.
14. Describe Yule-Furry process.
15. Establish the relation between Poisson process and binomial distribution

Turn over

16. Show that the state j is persistent if and only if $\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty$.

17. Consider the process $\{X(t), t \in T\}$ whose probability distribution is given by :

$$P\{X(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1, 2, \ldots \\ \dfrac{at}{1+at}, n = 0. \end{cases}$$

Examine whether the process $\{X(t), t \in T\}$ is stationary.

18. Define a Renewal process in discrete time and establish the relation between F (s) and P (s).

19. Describe equilibrium renewal process and show that Poisson process is an equilibrium renewal process.

20. What are the characteristics of a Queuing model?

20. Derive the mean and variance of number of objects in a Galton-Watson branching process.

21. Consider three state Markov chain having transition probability matrix :

$$P = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$. Show that P is ergodic and obtain the stationary distribution.

22. Show that the renewal function of a renewal process $\{N(t), t \geq 0\}$ generated by $X_n \, (n = 1, 2, \ldots)$ is given by $E(N(t)) = \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} + 0(1)$, where $\mu (< \infty)$ and σ^2 are the mean and the variance of X_n.

23. Describe gambler's ruin problem and find the probability of his ultimate ruin in the long run.

24. Describe a semi-Markov process, with an example.

(8 × 2 = 16 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Derive the probability distribution of Poisson process by stating its postulates.

26. Show that in an irreducible chain all states are of the same type. Also substantiate this concept through an example.

27. Describe the G/G/l Queuing model.

28. Show that Poisson process as a renewal process.

(2 × 4 = 8 weightage)

D 11247 (Pages : 3) Name.............................

Reg. No...........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3E 02—OPERATIONS RESEARCH—I

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. 1 Define the following (i) basic solution ; (ii) basic feasible solution ; (iii) optimum solution ; and (iv) slack variables.

2 State the limitations of Simplex method.

3 Solve graphically the following LPP :

$$\text{Minimize } Z = 20x_1 + 10x_2$$

$$\text{subject to } \quad x_1 + 2x_2 \leq 40$$
$$3x_1 + x_2 \geq 30$$
$$4x_1 + 3x_2 \geq 60$$
$$x_1, x_2 \geq 0.$$

4 Outline Vogel's technique of finding a basic feasible solution for a transportation problem.

5 Define degeneracy in a transportation problem and briefly explain the method of solving it.

6 Explain the parametrization of cost vector in a LPP.

7 How do you use Gantt Chart for solving sequencing problems ?

8 Show that every two person zero sum game with mixed strategies has a solution.

(4 × 4 = 16 marks)

Section B

Answer **either** *(A)* **or** *(B) of all questions.*
Each question carries 16 marks.

II. A. (a) Given an extreme point solution of a LPP, explain how we determine a new extreme point solution in a computationally efficient manner.

Turn over

(b) What is meant by degeneracy in a LPP ? Give an outline of perturbation techniques to resolve it.

(8 + 8 = 16 marks)

Or

B. (a) State and prove duality theorem.

(b) Write down the dual-simplex algorithm. Describe how it differ from the Simplex method.

(8 + 8 = 16 marks)

III. A. (a) Discuss the method of obtaining the optimum solution in a transportation problem. What is the procedure in the case of degeneracy ?

(b) What is an assignment problem ? How do you deal with the assignment problems, where (i) the objective function is to be maximized ; (ii) Some assignments are prohibited ?

(8 + 8 = 16 marks)

Or

B. (a) What is sensitivity analysis ? Discuss sensitivity analysis in a LPP, where an additional equality constraint is added.

(b) How will you solve the sequencing of n jobs on three machines ?

(8 + 8 = 16 marks)

IV. A. (a) Distinguish between Gomory's cutting plane method and branch and bound method for solving an integer programming problem.

(b) What is mean by zero-one programming ? Discuss some applications it.

(10 + 6 = 16 marks)

Or

B. Solve the following integer linear programming problem optimally :

Maximize $Z = 6x_1 + 8x_2$

Subject to

$$4x_1 + 5x_2 \leq 22$$
$$5x_1 + 8x_2 \leq 30$$
$$x_1, x_2 \geq 0 \text{ and integers.}$$

(16 marks)

V. A. (a) Explain the Maximin and Minimax principle used in game theory. Let (a_{ij}) be the payoff matrix for a two-person zero-sum game. If $\underline{v}$ denotes the maximin value and $\overline{v}$ the minimax value of the game, then show that $\overline{v} \geq \underline{v}$.

(b) Show that every two-person zero-sum game with mixed strategies has a solution.

(8 + 8 = 16 marks)

Or

B. (a) Explain the theory of dominance in the solution of rectangular games. Illustrate with examples.

(b) Explain the graphical method for solving a $2 \times n$ and $m \times 2$ games.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 11245 (Pages : 2) Name.......................................

Reg. No...................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 13—STOCHASTIC PROCESS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) When do you say that a process is weakly stationary ?

(b) Define one step transition probability matrix of a Markov chain, give an example.

(c) Describe an ergodic chain.

(d) What are the postulates of a Poisson process ?

(e) Define Compound, Poisson process and non-homogenous Poisson Process.

(f) What is offspring distribution with regard to a branching process ?

(g) Explain a renewal process.

(h) Explain random walk process.

(4 × 4 = 16 marks)

Part B

Answer **either** *part-A* **or** *part-B.*
Each question carries 16 *marks.*

II. A) (a) Explain four types stochastic processes based on nature of state space and index set. Give one example for each.

(b) Define transient state and recurrent state. Show that all the states of an irreducible finite chain are positive recurrent.

B) (a) Define Markov chain. Prove that a Markov chain is completely determined by its initial distribution and one step TPM.

(b) State and prove the Chapman - Kolmogorov Equations.

III. A) (a) Define Poisson process. Derive the steady state distribution of the Poisson process.

(b) Let $\{N(t)\}$ is a Poisson process with parameter λ. Suppose that each occurrence of the events has a constant probability p of being recorded independently. If $\{M(t)\}$ is the number of events being recorded in an interval of length t. Then show that $\{M(t)\}$ is a Poisson process with parameter λ p.

Turn over

B) (a) Show that a stochastic process $\{N(t)\}$ is a Poisson process iff its inter-arrival time distribution is Poisson.

(b) Show that states of one dimensional symmetrical random walk are recurrent.

IV. A) (a) Derive Kolmogrov Equations for continuous time Markov Process.

(b) Derive the recurrence relation satisfied by the probability generating function of a discrete branching process.

B) (a) Define a birth and death processes. Derive the forward Kolmogorov differential equation satisfied by the process.

(b) Derive the recurrence relation satisfied by the probability generating function of a discrete branching process.

V. A) (a) Explain how Brownian motion process can be obtained from a random walk through a limiting process.

(b) Describe renewal function and renewal equation, find the renewal function if the interval time distribution is Uniform [0, 1].

B) (a) State and prove elementary, renewal theorem.

(b) Explain the Characteristics of an M/M/1 Queueing system. Obtain its steady state system size distribution.

($4 \times 16 = 64$ marks)

D 11244 (Pages : 2) Name.............................

Reg. No..........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 12—MULTIVARIATE ANALYSIS

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Define Multivariate normal distribution. If X $N_p(\mu, \Sigma)$, find the distribution of Y = CX, where C is a non-singular matrix of order p.

(b) If X ~ $N_p(\mu, \Sigma)$ derive the characteristic function of X.

(c) Explain the hypothesis test concerning the mean vector μ of $N_p(\mu, \Sigma)$.

(d) State multivariate central limit theorem.

(e) State and prove the additive property of Wishart distribution.

(f) Explain the concept of principal components.

(g) Outline the classification problem.

(h) Explain canonical variates and canonical correlations.

($4 \times 4 = 16$ marks)

Part B

Answer **either** *(A)* **or** *(B) of all questions.*
Each question carries 16 *marks.*

II. A. (a) If X ~ $N_p(\mu, \Sigma)$, then show that $Q = (X-\mu)^T \Sigma^{-1}(X-\mu) \sim \chi_{(p)}$.

(b) If X ~ $N_p(\mu, \Sigma)$, prove that the distribution of the any sub vector of X is also multivariate normal.

B. (a) If $X = \left(X^{(1)} \cdot X^{(2)}\right) \sim N_p(\mu, \Sigma)$. Obtain the conditional distribution of $X^{(1)}$ given $X^{(2)}$.

(b) Let X ~ $N_p(\mu, \Sigma)$, obtain the MLE's of μ and Σ.

III. A. (a) Derive the distribution of Hotelling T^2 statistic.

(b) Define likelihood ratio test. Derive the test criterion to test the hypothesis that mean vectors of two multivariate normal populations are equal when they have same unknown covariance matrix.

Turn over

B. (a) Prove that Wishart distribution is a generalization of $\sigma^2\chi^2$ distribution.

(b) Derive the characteristic function of Wishart distribution.

IV. A. (a) Define sample generalized variance. Let $X_1, X_2, \ldots, X_N$ be a random sample from $N_p(\mu, \Sigma)$. Obtain the distribution of the sample generalized variance.

(b) If $X \sim N_p(0, 1)$. Show that X^TAX and X^TBX are independent if, and ony if $AB = 0$.

B. (a) Let X_1, X_2, X_n are independent random variables such that $X_1 \sim N_p(\mu, \Sigma)$, $i = 1, 2, \ldots, n$. Show that the mean vector $\overline{X}$ and SP matrix A are independent.

(b) Describe multivariate Fisher-Behren problem. Derive the test statistic as a T^2 statistic.

V. A. (a) Explain Bayes classification rule. Derive the linear discriminant function for classifying an observation between two multivariate normal distributions.

(b) Derive the relation between principal components and the eigen vectors of the variance covariance matrix.

B. (a) Explain the concept of principal components.

(b) Explain the iterative procedure to calculate sample principal components.

(4 × 16 = 64 marks)

D 11243 (Pages : 2) Name.............................

Reg. No...........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 11—STATISTICAL INFERENCE—II

(2019 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Define most powerful test.

(b) State and prove Neyman-Pearson lemma.

(c) Define unbiased and invariant tests.

(d) Define ASN function. Explain its uses.

(e) Define locality most powerful test.

(f) Explain generalized likelihood ratio test.

(g) State fundamental identity of SPRT.

(h) Define Kendall's tau.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *(A)* **or** *(B) of all questions.*
Each question carries 16 *marks.*

II. A. (a) Define power function. The consumption of electricity every day in a small town is having a probability function $f(x,\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$. Determine the significance level and power if $H_0 : \theta = 2000$ kw is rejected against $H_0 : \theta = 2700$ kw if the consumption of a randomly selected day is greater than 2650 kW. Obtain the power function and draw the power curve.

(b) Define MLR Property. Prove that U(0, θ) has MLR in $X_{(n)}$.

B. (a) Define UMP test. Obtain the UMP test for testing $H_0 : M \leq M_0$ against $H_1 : M > M_0$ based on a single observation from hypergeometric distribution with p.m.f.

$$f(x, M) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, x = 0, 1, 2, \ldots, M.$$

(b) State and prove Generalized Neyman-Pearson Lemma.

III. A. (a) Define likelihood ratio test. Obtain the asymptotic distribution of the likelihood ratio test statistic.

(b) Obtain the likelihood ratio test for testing the equality of means of two normal populations with equal variances.

B. (a) Define UMP test. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(0, \sigma^2)$. Obtain UMP test for testing $H_0 : \sigma > \sigma_0$ against $H_1 : \sigma \leq \sigma_0$.

(b) Briefly explain Union-intersection and Intersection-Union tests.

IV. A. (a) Explain Wilcoxon signed rank test.

(b) Describe Kolmogrov-Simrnov test.

B. (a) Explain Mann-Whitney Wilcoxon test.

(b) Define median test. Derive its null distribution.

V. A. (a) Define Sequential probability ratio test and derive the boundary values of it.

(b) Derive the expression for OC function corresponding to the sequential probability ratio test.

B. (a) Prove that the sequential probability ratio test terminates with probability one.

(b) Obtain the ASN function corresponding to the sequential probability ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1, \mu_0 < \mu_1$ based on observations from $N(\mu, \sigma^2)$ at strength (α, β), where σ^2 is known.

($4 \times 16 = 64$ marks)

D 11241 (Pages : 2) Name..................................

Reg. No..............................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 13—STOCHASTIC PROCESS

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Define state space of a stochastic process. Give an example for a continuous state space stochastic process.

(b) Define a Markov chain. Give one example.

(c) What do you meant by a counting process ?

(d) Differentiate between homogenous and non-homogenous Poisson processes.

(e) Define semi Markov process

(f) Briefly explain insurers ruin problem.

(g) Briefly explain the important characteristics of a queue.

(h) Define weakly stationary process. Give one example.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *part (a)* **or** *part (b) of all questions.*
Each question carries 16 *marks.*

II. (a) (i) How will you classify a Markov stochastic process ?

(ii) Explain Gamblers ruin problem and establish the conditions for the existence of stationary distribution.

Or

(b) (i) Show that the transition probability matrix and the initial probability vector describes a Markov chain completely.

(ii) State and prove the Chapman - Kolmogorov equation.

Turn over

III. (a) (i) Derive the distribution of Poisson process stating all its postulates.

(ii) If $\{X(t)\}$ is a Poisson process, find the auto correlation coefficient between $\{X(t)\}$ and $X(t+s)\}$.

Or

(b) (i) Derive the pgf of non-homogeneous Poisson process.

(ii) Obtain the difference differential equations of a birth and death process.

IV. (a) (i) Establish the renewal equation.

(ii) State and prove the elementary renewal theorem.

Or

(b) (i) Describe the concept of stopping time and establish the Wald's equation

(ii) Define the terms residual life time and current life time. How does it characterize Poisson process.

V. (a) (i) Obtain the steady state probability distribution of an M/M/1 queue with finite system size.

(ii) Obtain the steady state probability distribution of an G/M/l model queue.

Or

(b) (i) Define Brownian Bridge. If $\{X(t), t \geq 0\}$ is a Brownian motion process, show that $\{Z(t), 0 \geq t \geq 1\}$ is a Brownian Bridge process when $Z(t) = X(t) - tX(1)$.

(ii) Derive the distribution of maximum of a Wiener Process.

($4 \times 16 = 64$ marks)

D 11240 (Pages : 2) Name..............................

Reg. No..........................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 12—MULTIVARIATE ANALYSIS

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. 1 If $\underline{X} \sim N_p(\theta, \Sigma)$, then find the distribution of $l'\underline{X}$, where l is a vector of order $p \times 1$.

2 Define (i) partial correlation and (ii) Multiple correlation.

3 If $V_1 \sim W_p(I, n_1)$ and $V_2 \sim W_p(I, n_2)$ then find the distribution of $\frac{|V_1|}{|V_2|}$.

4 Define generalized variance. Obtain its hth moment.

5 Define Mahalanobis distance.

6 Obtain the relationship between Mahalanobs D^2 statistic and Hotelling's T^2 statistic.

7 Explain the classification problem with a suitable example.

8 What are principle components and discuss its properties.

(4 × 4 = 16 marks)

Section B

Answer **either** *(A)* **or** *(B) of all questions.*
Each question carries 16 marks.

II. A. (a) If $\underline{X} \sim N_p(\theta, \Sigma)$, then derive the distribution of $X'\Sigma^{-1}X$.

(b) If $\underline{X} \sim N_p(0, I)$, then state and prove a necessary and sufficient condition for $X'AX$ to have a Chi-square distribution.

Or

B. (a) If $\underline{X} \sim N_p(\theta, \Sigma)$, then show that $\underline{X}'A\underline{X}$ and $\underline{X}'B\underline{X}$ are independent if and only if $A\Sigma B = 0$.

Turn over

(b) Obtain the expression for partial correlation between X_1 and X_2, when $(X_1, X_2, \ldots, X'_n) \sim N_p(0, \Sigma)$.

III. A. (a) Derive the test criterion to test the hypothesis that mean vector of a multivariate normal population is a null vector, when the dispersion matrix is known.

(b) Derive the characteristic function of Wishart distribution. State and prove additive property of Wishart distribution.

Or

B. (a) Show that MLE's of θ and Σ in $N_p(\theta, \Sigma)$ are independent.

(b) Derive the likelihood ratio test to test the equality of means of two multivariate normal population when the variance covariance matrices are known.

IV. A. (a) Define Mahalanobis' D^2 statistic and obtain its distribution.

(b) Derive the likelihood ratio test for test the equality of dispersion matrices of two multivarate normal population.

Or

B. (a) Outline a suitable test procedure for testing the hypothesis that the dispersion matrix of a normal vector is proportional to a given matrix.

(b) Derive the likelihood ratio test to test the equality of means of two multivariate normal populations when the variance covariance matrices are equal but unknown.

V. A. (a) Describe the iterative procedure to calculate sample principal components.

(b) Describe the problem of classification into one of several multivariate normal population, when the costs of misclassification are equal.

Or

B. Suppose that in a classification problem that two underlying populations are normal distribution with unknown parameters μ_1, μ_2 and Σ (the common dispersion matrix). Obtain the classification region such that the probabilities of two misclassification are equal, given large samples from both the populations.

($4 \times 16 = 64$ marks)

D 11239 (Pages : 3) Name.....................................

Reg. No.....................................

THIRD SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 3C 11—STATISTICAL INFERENCE – II

(2013 Admissions)

Time : Three Hours Maximum : 80 Marks

Part I

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (a) Define test function. Distinguish between randomized and non randomized tests.

(b) Let p be the probability that a coin will fall head in a single toss. To test $H_0 : p = 1/2$ against $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and power of the test.

(c) Explain families of distributions with monotone likelihood ratio property.

(d) Define (i) UMP test ; (ii) unbiased test and (iii) UMP unbiased test.

(e) Describe one sample sign test.

(f) Explain the sequential probability ratio test procedure.

(g) Define (i) Kendall's tau ; and (ii) Spearmans rank correlation co-efficient.

(h) Write short notes on advantages and disadvantages of non parametric tests.

(4 × 4 = 16 marks)

Part II

Answer **either** *Part* (A) **or** *Part* (B) *of all questions.*
Each question carries 16 *marks.*

II. (A) (i) State and prove Neymann Pearson lemma.

(ii) Obtain most powerful test for testing $H_0 : \sigma^2 = \sigma_0{}^2$ against the alternative $H_1 : \sigma^2 = \sigma_1{}^2$ in case of normal population $N(\mu, \sigma^2)$ based on a random sample of size n from the population.

(8 + 8 =16 marks)

Or

Turn over

(B) (i) Prove that UMP test of one sided tests always exists for families of distributions possessing monotone likelihood property.

(ii) Let X be a random variable having probability function :

$$P(X=k)=\frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}} \quad \text{for } k = 0, 1, 2\ldots \min(n, M).$$

Examine whether there exists UMP for testing $H_0 : M \leq M_0$ against $H_1 : M > M_0$

(iii) Show that If a sufficient statistic T exists for the family of distributions, then Neyman-Pearson MP test is a function of T.

(9 + 4 + 3 =16 marks)

III. (A) (i) Define (a) α similar test ; (b) UMP α similar test ; (c) Invariant test ; (d) maximal invariant.

(ii) Show that if a power function is continuous, then UMP α similar test is UMP unbiased provided its size is α.

(iii) Find UMP unbiased test for testing $H_0 : \mu \leq 0$ against $H_1 : \mu > 0$ based on a random sample of size n from $N(\mu,1)$.

(6 + 5 + 5 = 16 marks)

Or

(B) (i) Explain the likelihood ratio property. State the asymptotic properties of likelihood ratio test.

(ii) Construct likelihood ratio test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ based on a random sample of size n from a population with pdf

$$f(x,\theta,\lambda)=\frac{1}{\lambda}\exp\left(\frac{-(x-\theta)}{\lambda}\right), \theta < x < \infty, \text{ when } \lambda > 0 \text{ is known.}$$

(6 + 10 = 16 marks)

IV .(A) (i) Define Kolmogrov-Smirnov statistics D_n^+, D_n^-, and D_n. Show that they are distribution free for any continuous cumulative distribution function F.

(ii) Explain Wilcoxon signed rank test.

(8 + 8 =16 marks)

(B) (i) Write short notes on the following :

(a) Mann-Whitney U statistic.

(b) Kolmogrov-Smirnov two sample test.

(ii) Explain the hypothesis testing for a population quantile.

(10 + 6 =16 marks)

V. (A) (i) Let X have the distribution $f(x,p) = p^x (1-p)^{1-x}$; $x = 0, 1$; $0 < p < 1$. Construct SPRT for testing $H_0 : p = p_0$ against $H_1 : p = p_1$ for $p_1 > p_0$.

(ii) For the SPRT with stopping bounds A and B, where A < B and strength (a, b) prove that $A \le \frac{1-\beta}{\alpha}$ and $B \ge \frac{\beta}{1-\alpha}, 0 < \alpha < 1, 0 < \beta < 1$.

(8 + 8 =16 marks)

Or

(B) (i) Show that SPRT terminates with probability one.

(ii) Construct SPRT for an exponential distribution with mean θ for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ for $\theta_1 > \theta_0$.

(10 + 6 = 16 marks)