D 13172 (Pages : 2) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 1C 04—PROBABILITY THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. Give an example to show that a field need not be a sigma field. What do you mean by a probability measure induced by a random variable X ?
2. Obtain the characteristic function of a standard Laplace distribution.
3. Prove that if $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then $X_n + Y_n \xrightarrow{P} X + Y$.
4. Let $X_1, X_2, \ldots$ be a sequence of independent random variables with p. m. f. $p(X_n = 0) = 1 - \frac{1}{n^\alpha}$ and $p(X_n = \pm n) = \frac{1}{2n^\alpha}$, for $\alpha > 1$.

   Examine whether $X_n \xrightarrow{as} 0$.
5. State Borel Cantelli lemma.

Turn over

6. State Helly-Bray theorem. What is its significance ?

7. State Lindberg-Feller theorem on central limit theorem.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Prove that a sigma field is a monotone field and conversely.

9. Define distribution function. Show that a distribution function is non-decreasing and right continuous.

10. State and prove Cr inequality.

11. Show that Characteristic function is uniformly continuous over R.

12. Define convergence in probability and convergence in the rth mean. Show that convergence in the r^{th} mean implies convergence in probability.

13. State and prove Kolmogorov three series theorem.

14. For the following sequence of independent random variables does the WLLN hold :

    a) $P\left\{X_k = \pm 2^k\right\} = 1/2$.

    b) $P\left\{X_k = \pm\sqrt{k}\right\} = 1/2$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Show that $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{L} X$. By an example show that converse need not be true.

16. State continuity theorem on characteristic function.

17. a) State and prove Liapunov Central limit theorem.

    b) Show that Liapunov condition implies Lindberg-Feller condition.

18. Establish Kolmogorov strong law of large numbers for a sequence of independent random variables.

(2 × 5 = 10 weightage)

D 13171 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 1C 03—DISTRIBUTION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. If $X \sim B(n, p)$, Prove that $k_{r+1} = p.(1-p)\frac{d}{dp}k_r$, where k_r, is the r^{th} cumulant.

2. Obtain the moment generating function of the power series distribution.

3. If $X_1, X_2, \ldots\ldots, X_n$ are iid random variables following the Cauchy distribution, obtain the distribution of $\overline{X}$.

4. Let $F(x)$ be the distribution function of a continuous random variable. Show that $Y = F(x) \sim U[0,1]$.

5. Define noncentral χ^2 distribution. Find its mean and variance.

Turn over

6. State a the lack of memory property. Identify two distributions satisfying this property.

7. If $X_1, X_2, \ldots\ldots\ldots, X_n$ are iid random variables following the exponential distribution. Find the distribution of $X_{(1)} = \text{Min}\left(X_1, X_2, \ldots\ldots\ldots, X_n\right)$ and $X_{(n)} = \text{Max}\left(X_1, X_2, \ldots\ldots\ldots, X_n\right)$.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Derive the recurrence relation for the moments of the binomial distribution. Find the values of β_1 and β_2.

9. Obtain the poisson distribution as a limiting case of the negative binomial distribution.

10. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample of size n from Cauchy distribution. Find the distribution of $\overline{X}$.

11. Derive the moment generating function of the Normal distribution.

12. If X and Y are independent Gamma variates with parameters α and β. Find the distribution of $V = \dfrac{X}{Y}$.

13. Define Pareto distribution. If Y is a random variable following Pareto distribution, find the distribution of $X = \log Y$.

14. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be iid with distribution function $F(x) = x^{\alpha}$, if $0 < x < 1, \alpha > 0$. Show that $\dfrac{X_{(i)}}{X_{(n)}}, i = 1, 2, \ldots\, n-1$ and $X_{(n)}$ are independent.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Let X and Y be jointly distributed with probability density function :

$$f(x, y) = \frac{1 + xy}{4}, \quad |x| < 1, |y| < 1.$$

Examine the independence of (i) X and Y ; and (ii) X^2 and Y^2.

16. Define non-central t distribution. Derive the probability density function of it.

17. Let X_1, X_2 be iid $U[0, 1]$ random variables. Let $Y_1 = X_1X_2$ and $Y_2 = \frac{X_1}{X_2}$, find the probability density function's of Y_1 and Y_2.

18. Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be iid with probability density function $f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, x > 0$. Show that $X_{(1)}, X_{(2)} - X_{(1)}, \ldots\ldots, X_{(n)} - X_{(n-1)}$ are independent.

(2 × 5 = 10 weightage)

D 13170 (Pages : 2) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 1C 02—ANALYTICAL TOOLS FOR STATISTICS-II

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Each question carries 2 weightage.

1. Explain linear dependency and independency of Vectors over a field.
2. Define vector space and give one example.
3. Define orthogonal and orthonormal basis.
4. Define Hermition and skew Hermition matrices. Give examples.
5. Define algebraic and geometric multiplicities.
6. State rank -nullity theorem.
7. Show taht g-inverse is not unique.

($4 \times 2 = 8$ weightage)

Part B

Answer any **four** *questions.*
Each question carries 3 *weightage.*

8. Examine whether the following vectors are linearly independent.

$$\{(1,1,0),(1,3,2),(4,9,5)\}$$

9. Define sub space. Show that, the intersection of any number of subspaces of a vector space V is also a subspace of V.

10. State and prove Cayley-Hamilton theorem.

11. Prove that the eigen values of an orthogonal matrix are ± 1.

12. Obtain the orthogonal basis of the set of vectors $\{(1,1,1), (1,2,4), (-2,3,7)\}$.

13. Define Moore-Penrose g inverse. Show that it is unique.

14. Solve the system of equations using Gauss elimination method.

$$3x + y - z = 3$$
$$2x - 8y + z = -5$$
$$x - 2y + 9z = 8$$

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

15. Let U and W be subspaces of a vector space V. Show that
    (i) U + W is a subspace of V.
    (ii) U + W is the smallest subspace containing U and W.
    (iii) Find the Basis and dimension of the subspace W of R^3, where $W = \{(a, b, c) : a + b + c = 0\}$

16. Define g-inverse of a matrix. Find the g-inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 7 & 6 \\ 7 & 4 & 3 & 1 \\ 0 & 2 & 0 & 5 \end{bmatrix}$

17. Explain spectral decomposition of a symmetric matrix.

18. (i) Show that a set of orthogonal vectors are linearly independent.
    (ii) Discuss Gram -Schmidt orthogonalization.

(2 × 5 = 10 weightage)

D 13169 (Pages : 2) Name.......................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

(CBCSS)

Statistics

MST 1C 01—ANALYTICAL TOOLS FOR STATISTICS-I

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer any **four** *questions.*
Weightage 2 for each question.

1. Examine whether the limit of the function $f(x,y)=\dfrac{x^4+y^4}{x^3+y^3}$ exists at the point (0, 0) along any straight line through the origin.
2. Define Riemann integral of a multivariate function.
3. Define the terms poles and singularities of a function.
4. Define Laplace transform and inverse Laplace transform of a function.
5. State Poisson integral formula.
6. Determine the nature and singularity of the function $\dfrac{\sin z}{z}$ at $z = 0$.
7. State Fourier integral theorem.

(4 × 2 = 8 weightage)

Part B

Answer any **four** *questions.*
Weightage 3 *for each question.*

8. Find the directional derivative of $x^2y+2yz-x$ at P(1, 1, 1) in a direction towards Q(2, –3, 2).

9. Examine the function $x^3+y^3-3x-12y+20$ for maximum and minimum.

10. Show that every analytic function satisfies Cauchy-Riemann equations. Also derive the same in polar form.

11. Classify the nature and singularity of the function $f(z)=\frac{z^2}{(z-2)^2}$ and find the residue at the singular point.

12. Integrate $\frac{1}{z^4-1}$ around the circle (i) $|z+1|=1$ (ii) $|z+3|=1$.

13. State and prove Residue theorem.

14. Find the Laplace transform of

    (i) $f(t)=(t-1)^3+\sin t$. (ii) $\sin^2 t$.

(4 × 3 = 12 weightage)

Part C

Answer any **two** *questions.*
Weightage 5 *for each question.*

15. (i) Investigate the continuity of the function

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}; & \text{if } (x,y)\neq(0,0)\\ 0 \; ; & \text{if } (x,y)=(0,0)\end{cases}$$

    (ii) Find the directional derivative of $f(x,y)=x^2y+2yz-x$, at P(1, 1, 1) in a direction towards Q (2,–3, 2).

16. State and prove Laurent's theorem.

17. Find the Fourier series expansion of $f(x)=x+x^2;-\pi<x<\pi$.

18. (i) Obtain the Laurent's series expansion of

$$f(z)=\frac{2}{(z-1)(z-3)} \text{ valid in } 0<|z-1|<2.$$

    (ii) Evaluate $\int_0^\infty \frac{x^2}{x^4+5x^2+6}\,dx$.

(2 × 5 = 10 weightage)

D 12316 (Pages : 2) Name.....................................

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 1 C 05—SAMPLING THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*

I. (i) Mention situations in which sampling is the only resort.

(ii) State the limitations of sampling.

(iii) State the difference between simple random sampling and PPS sampling procedures.

(iv) Explain cumulative total method.

(v) When do you prefer ratio method of estimation ?

(vi) Why the regression estimator is biased ?

(vii) What are the disadvantages of systematic sampling ?

(viii) What do you mean by a cluster ? How do you specify them ?

(4 × 4 = 16 marks)

Section B

Answer **all** *questions.*

II. (A) (a) Obtain an unbiased estimator of the population proportion in case of simple random sampling without replacement. Also obtain its variance.

(b) Explain stratified random sampling procedure.

(10 + 6 = 16 marks)

Or

(B) (a) State the principles of stratification.

(b) With usual notations, show that,

$$V_{opt} \le V_{prop} \le V_{SR}.$$

(6 + 10 = 16 marks)

Turn over

III. (A) (a) Obtain the gain due to PPS sampling with replacement over simple random sampling.

(b) What is Murthy's unordered estimator ? Show that it is unbiased in PPS sampling without replacement. Obtain its sampling variance.

(8 + 8 = 16 marks)

Or

(B) (a) Obtain Yates - Grundy estimator.

(b) Explain ITPS sampling.

(6 + 10 = 16 marks)

IV. (A) (a) Obtain the expression for bias of the ratio estimator.

(b) Derive the first order approximation of the variance of $\hat{R}$ under simple random sampling without replacement.

(6 + 10 = 16 marks)

Or

(B) Compare the regression estimator with the mean per unit and ratio estimators and comment on the preference of the ratio and regression estimators.

(10 + 6 = 16 marks)

V. (A) (a) Obtain an estimator of mean and its variance under equal cluster sampling.

(b) Derive the relative efficiency of cluster sampling over simple random sampling.

(6 + 10 = 16 marks)

Or

(B) (a) Compare systematic sampling with simple random sampling.

(b) Explain multi-stage sampling procedures with its advantages and disadvantages.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 12315 (Pages : 2) Name.....................................

Reg. No.................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 1C 04—DISTRIBUTION THEORY

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Derive the M.G.F of binomial distribution.

(ii) Write a short note on multinomial distribution.

(iii) Explain the form of Pearson system of distribution.

(iv) Explain any two forms of transforming distribution.

(v) Define order statistic and find the pdf of n^{th} order statistic.

(vi) Explain how to obtain the marginal distribution from the joint distribution of vector of random variables with an illustration.

(vii) Write the applications of chi-square distribution.

(viii) Derive the mean of non-central F distribution.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *part -A* **or** *part - B of* **all** *questions.*
Each question carries 16 *marks.*

II. A. a) Derive the first two moments of negative binomial distribution.

b) Define logarithmic series distribution and derive its mean and variance.

($8 + 8 = 16$ marks)

Or

B. a) Derive the mean and variance of discrete uniform distribution.

b) Define the power series distribution and derive its M.G.F.

($8 + 8 = 16$ marks)

Turn over

III. A. a) Show that ratio of two independent standard normal variable follows standard Cauchy distribution.

b) Derive the r^{th} moment of Laplace distribution.

(8 + 8 = 16 marks)

Or

B. a) Show that for lognormal distribution mean > median > mode.

b) Derive the basic statistical properties of bivariate normal distribution.

(8 + 8 = 16 marks)

IV. A. a) Derive the pdf of arbitrary order statistic from a random sample of size n.

b) Consider the bivariate negative binomial distribution with pmf

$$P(X = x, Y = y) = \frac{(x+y+k-1)!}{x!y!(k-1)!} p_1^x p_2^y (1-p_1-p_2)^k ; x, y = 0,1,2,\ldots,k \geq 1; p_1, p_2 \in (0,1),$$

Let U = X + Y and V = Y. Find : (i) the joint pmf of U and V. (ii) the marginal pmf of U.

(8 + 8 = 16 marks)

Or

B. a) Let (X, Y) follows bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ. Find (i) the marginal density of X and Y. (ii) The conditional probability distribution of Y/X.

b) Let $X_1, X_2, \ldots, X_5$ be the random variables from an exponential distribution with parameter λ = 2. Find the pdf of 2^{nd} and 4^{th} order statistic.

(8 + 8 = 16 marks)

V. A. a) Derive the pdf of non-central F distribution

b) Derive the characteristic function of student's t distribution.

(8 + 8 = 16 marks)

Or

B. a) Derive the pdf of non-central t distribution.

b) Explain the interrelation between t, F and chi-square distribution.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 12314 **(Pages : 3)** **Name..............................**

Reg. No..............................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 1C 03—PROBABILITY THEORY — I

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Distinguish between probability space and measure space.

(ii) Define empirical distribution function and mention its uses.

(iii) Define convex function and give an example.

(iv) Obtain the moment generating function of gamma variate.

(v) State the necessary and sufficient conditions for convergence in probability.

(vi) Define convergence in distribution and convergence in mean.

(vii) If A and B are independent events and $A \subseteq B$, show that either $P(A) = 0$ or $P(B) = 1$.

(viii) State Bochner's Theorem.

(4 × 4 = 16 marks)

Section B

Answer **either** *Part A* **or** *Part B of all questions.*
Each question carries 16 *marks.*

II. A (a) If X is a random variable, then show that $aX + b$ is also a random variable, where a and b are constants.

(6 marks)

(b) Define distribution function of a random variable X. State and prove any four properties of distribution function.

(10 marks)

Or

Turn over

B (a) Define Kolmogorov's axiomatic definition of probability. (6 marks)

(b) If $\{A_n : 1 \le n < \infty\}$ is any sequence of events, show that :

$$P(A_*) \le \lim_{n\to\infty} \inf P(A_n) \le \lim_{n\to\infty} \sup P(A_n) \le P(A^*),$$ where A^* is limit superior and A^* limit inferior of the sequence $\{A_n\}$ of events.

(10 marks)

III. A (a) Let X take the values $\{0, \pm 1, \pm 2, \ldots\}$ with $P[X = k] = p[X = -k] = (3/\pi^2)/k^2, k = 1, 2, \ldots$ and P [X = 0] = 0, Find E (X), if exists.

(8 marks)

(b) State basic inequality and deduce it to (i) Markov inequality ; and (ii) Chebyshev's inequality.

(8 marks)

Or

B (a) Find the moment generating function when the cumulative distribution function of X is $F(x) = 0, x < 0 : F(x) = 1 - ½\, e^{-x}, x \ge 0$.

(6 marks)

(b) If $E|X|^r < \infty$, then show that $E|X|^{r'} < \infty$ for $0 < r' \le r$ and EX^k exists and is finite for $k < r$, k be an integer.

(10 marks)

IV. A (a) Show that a sequence of random variables cannot converge in probability to two essentially different random variables. (8 marks)

(b) Define almost sure convergence. State and prove the necessary and sufficient condition for the convergence of $\{X_n\}$ almost surely.

(8 marks)

Or

B (a) If $f(x)$ is a continuous real-valued function and $X_n \xrightarrow{p} X$ prove that $f(X_n) \xrightarrow{p} f(X)$.

(8 marks)

(b) What is meant by convergence of $\{X_n\}$? Verify whether the sequence of independent variates $\{X_n\}$ defined with $P(X_n = 1) = 1/n, P(X_n = 0) = 1 - (1/n), n = 1, 2, 3, \ldots$ is convergent in mean.

(8 marks)

V. A (a) Let $(A_1, A_2, \ldots, A_m)$ and $(B_1, B_2, \ldots, B_n)$ be two sets of disjoint events. If eve

B_j are independent for every choice of i and j, show that $A\,(= \cup A_i)$ and $B(=$ independent (i = 1, 2, .. m ; j = 1 ,2, ... n).

(b) Establish Taylor series expansion of characteristic function.

Or

B (a) If X_1, X_2, ... are i.i.d. random variables with common mean μ and finite fo

moment, then show that $P\left(\lim_{n \to \infty} \frac{S_n}{n} = \mu\right) = 1$, where $S_n = X_1 + X_2 + \ldots + X_n$.

(b) Obtain the density function of the distribution whose characteristic fu $p(1 - q\exp(it))^{-1}, (p, q > 0, p + q = 1)$.

[4 × 16 =

B (a) Define Kolmogorov's axiomatic definition of probability. (6 marks)

(b) If $\{A_n : 1 \leq n < \infty\}$ is any sequence of events, show that :

$P(A_*) \leq \lim_{n \to \infty} \inf P(A_n) \leq \lim_{n \to \infty} \sup P(A_n) \leq P(A^*)$, where A^* is limit superior and A_* limit inferior of the sequence $\{A_n\}$ of events.

(10 marks)

III. A (a) Let X take the values $\{0, \pm 1, \pm 2, \ldots\}$ with $P[X = k] = p[X = -k] = (3/\pi^2)/k^2, k = 1, 2, \ldots$ and P [X = 0] = 0, Find E (X), if exists.

(8 marks)

(b) State basic inequality and deduce it to (i) Markov inequality ; and (ii) Chebyshev's inequality.

(8 marks)

Or

B (a) Find the moment generating function when the cumulative distribution function of X is $F(x) = 0, x < 0 : F(x) = 1 - ½\, e^{-x}, x \geq 0$.

(6 marks)

(b) If $E|X|^r < \infty$, then show that $E|X|^{r'} < \infty$ for $0 < r' \leq r$ and EX^k exists and is finite for $k < r$, k be an integer.

(10 marks)

IV. A (a) Show that a sequence of random variables cannot converge in probability to two essentially different random variables. (8 marks)

(b) Define almost sure convergence. State and prove the necessary and sufficient condition for the convergence of $\{X_n\}$ almost surely.

(8 marks)

Or

B (a) If $f(x)$ is a continuous real-valued function and $X_n \xrightarrow{p} X$ prove that $f(X_n) \xrightarrow{p} f(X)$.

(8 marks)

(b) What is meant by convergence of $\{X_n\}$? Verify whether the sequence of independent variates $\{X_n\}$ defined with $P(X_n = 1) = 1/n, P(X_n = 0) = 1 - (1/n), n = 1, 2, 3, \ldots$ is convergent in mean.

(8 marks)

V. A (a) Let $(A_1, A_2, \ldots, A_m)$ and $(B_1, B_2, \ldots, B_n)$ be two sets of disjoint events. If events A_i and B_j are independent for every choice of i and j, show that $A\,(= \cup A_i)$ and $B(= \cap B_j)$ are independent (i = 1, 2, .. m ; j = 1 ,2, ... n).

(8 marks)

(b) Establish Taylor series expansion of characteristic function. (8 marks)

Or

B (a) If $X_1, X_2, \ldots$ are i.i.d. random variables with common mean μ and finite fourth order moment, then show that $P\left(\lim_{n \to \infty} \frac{S_n}{n} = \mu\right) = 1$, where $S_n = X_1 + X_2 + \ldots + X_n$. (8 marks)

(b) Obtain the density function of the distribution whose characteristic function is $p(1 - q\exp(it))^{-1}, (p, q > 0, p + q = 1)$.

(8 marks)

[4 × 16 = 64 marks]

D 12313 **(Pages : 3)** **Name.....................................**

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA I C 02—MATHEMATICAL METHODS FOR STATISTICS—II

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 marks.

I. (i) Define monotone class of sets. What is meant by monotone class generated by any class of sets ?

(ii) Distinguish measure and outer measure.

(iii) Define convergence everywhere and uniform convergence of a sequence of measurable functions.

(iv) What are integrable simple functions ?

(v) State Fubini's theorem ? What is its significance ?

(vi) What are singular measures ?

(vii) Define linear combination of vectors and linear span.

(viii) State two equivalent ways of defining a basis of a vector space.

(4 × 4 = 16 marks)

Section B

Answer either Part A or Part B of **all** *questions.*
Each question carries 16 marks.

II. (A) (a) Prove that the set function μ is countably additive on a class of all bounded, left closed and right opened intervals.

(b) Let μ be a measure on a sigma field, A, and $A \in A$. If $\{A_n, n = 1,2,...\}$ is finite or infinite disjoint sequence of sets in A such that $\bigcup_{i=1}^{\infty} A_i \subset A$, prove that $\sum_{i=1}^{\infty} \mu(A_i) \leq \mu(A)$.

(8 + 8 = 16 marks)

Or

Turn over

(B) (a) Define sigma field. Show that a measure defined on a sigma field is monotone and subtractive.

(b) Define : Borel set, Lebesque measure and Lebesgue-Stieltjes measure. Show that every countable set is a Borel set of measure zero.

(8 + 8 = 16 marks)

III. (A) (a) Show that a mean fundamental sequence of integrable functions is fundamental in measure.

(b) State and prove Egoroff's theorem.

(8 + 8 = 16 marks)

Or

(B) (a) State Lebesgue's bounded convergence theorem.

(b) If f and g are integrable functions such that $f \geq g$ almost everywhere, show that $\int f d\mu \geq \int g d\mu$.

(8 + 8 = 16 marks)

IV. (A) (a) If μ is a signed measure and $\{A_n\}$ is a disjoint sequence of measurable sets such that $\left|\mu\left(\bigcup_{n=1}^{\infty} A_n\right)\right| < \infty$, prove that $\sum_{n=1}^{\infty} \mu(A_n)$ is absolutely convergent.

(b) State Radon-Nikodym theorem. State the properties relating to Radon-Nikodym derivatives.

(8 + 8 = 16 marks)

Or

(B) (a) Show that every section of a measurable function is a measurable function.

(b) If (X, S) and (Y, T) are measurable spaces, show that the product space (X × Y, S × T) is a measurable space.

(8 + 8 = 16 marks)

V. (A) (a) What are the axioms to be satisfied by a non-empty set to become a vector space ?

(b) Define linear span and spanning set. Write an algorithm for finding the basis for a subspace of K^n spanned by a given vector.

(8 + 8 = 16 marks)

Or

(B) (a) Define linear dependence and linear independence of vectors in a vector space. Determine whether the vectors $u = (1, 2, -3)$ and $v = (4, 5, -6)$ are linearly dependent.

(b) Let $F : R^2 \to R^2$ be the linear operator defined by $F(x, y) = (2x + 3y, 4x - 5y)$. Find the matrix representation of F relative to the basis $S = \{u_1, u_2\} = \{(1, 2), (2, 5)\}$.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]

D 12312 (Pages : 3) Name.....................................

Reg. No..................................

FIRST SEMESTER P.G. DEGREE EXAMINATION, NOVEMBER 2021

(CCSS)

Statistics

STA 1C 01—MATHEMATICAL METHODS FOR SATISTICS—I

(2019 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Section A

Answer any **four** *questions.*
Each question carries 4 *marks.*

I. (i) Define lower and upper Riemann - Stieltjes sums.

(ii) Establish the relationship between beta and gamma functions.

(iii) Define boundedness of a sequence. Give an example to each of the following :

(a) a sequence which is bounded and

(b) a sequence which is not bounded.

(iv) State and prove a necessary condition for convergence of a series.

(v) Define extreme point and extreme value of a function of several variables. State the necessary condition for a function to have an extreme value.

(vi) State the sufficient condition for a function $f(x, y)$ to be continuous at (x_0, y_0).

(vii) Distinguish algebraic and geometric multiplicity of characteristic roots.

(viii) Show that a matrix and its transpose have the same characteristic polynomial.

($4 \times 4 = 16$ marks)

Section B

Answer **either** *part-A* **or** *part-B of* **all** *questions.*
Each question carries 16 *marks.*

II. A. (a) If f is RS integrable on $[a, b]$ with respect to a monotonically non-decreasing function α on $[a, b]$, derive the bounds for $\int_a^b f(x)\, d\alpha(x)$.

(b) Define gamma function. Show that $\Gamma(n) = (n-1)!$.

($8 + 8 = 16$ marks)

Or

Turn over

B. (a) What is meant by convergence of integrals ? Examine the convergence of $\int_0^1 \frac{1}{\sqrt{1-x}}\,dx.$

(b) Define a beta function. Show that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = B(m,n),$ for $m,n > 0.$

(8 + 8 = 16 marks)

III. A. (a) Define limit of a sequence. Prove that every bounded sequence has a limit point.

(b) Show that geometric series $\sum_{n=0}^{\infty} r^n$ converges for $r < 1$ and diverges to ∞ for $r \geq 1.$

(8 + 8 = 16 marks)

Or

B. (a) State and prove a necessary and sufficient condition for a monotonically increasing sequence to be convergent.

(b) Discuss the convergence of the series $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n!}.$

(10 + 6 = 16 marks)

IV. A. (a) Show that the partial derivative of the following function exist at (0, 0), but the function is not continuous at (0, 0) :

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(b) Distinguish between explicit and implicit functions.

(10 + 6 = 16 marks)

Or

B. (a) Find the optima of the function defined by $f(x,y) = x^3 + y^3 - 3x - 12y + 20.$

(b) Define continuity, differentiability and partial derivatives of a function of several variables.

(8 + 8 = 16 marks)

V. A. (a) Define similar matrices. Show that determinants of similar matrices are equal.

(b) Define characteristic polynomial. Show that every matrix is a root of its characteristic polynomial.

(6 + 10 = 16 marks)

Or

B. (a) Show that rank(AB) $\leq$ rank (A) and rank (AB) $\leq$ rank (B), where A and B are two matrices.

(b) State the condition for the positive definiteness of a quadratic form. Examine whether $Q(x, y, z) = x^2 + 2y^2 - 4xz - 4yz + 7z^2$ is positive definite.

(8 + 8 = 16 marks)

[4 × 16 = 64 marks]