D 14111 **(Pages : 2)** **Name....................................**

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2021

(CBCSS)

Mathematics with Data Science

MTD 3E 02—MACHINE LEARNINGS ESSENTIALS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* ***all*** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. What do you mean by a learning system ?
2. Explain the concept of reinforcement learning.
3. List four machine learning use cases.
4. Describe the relevance of threshold in logistic regression.
5. What are support vectors in SVMs ?
6. What do you mean by "curse of dimensionality" ?
7. Explain support and confidence in Association rule mining.
8. What is the importance of rewards in Reinforcement Learning ?

(8 × 1 = 8 weightage)

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Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. Explain the concept and goals of unsupervised learning.
10. What is the significance α and β of regression line $y = \alpha x + \beta$?
11. Explain the concept of bagging.

Unit II

12. Explain how entropy and information gain are useful in Decision tree learning.
13. Differentiate between precision and recall with example.
14. Explain cosine similarity used for collaborative filtering.

Unit III

15. Explain the apriori algorithm.
16. What is the exploration vs. exploitation dilemma ?
17. What is counting regret ?

(6 × 2 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. Explain types of machine learning and different models in each type with proper examples.
19. Derive the least square estimator for multiple linear regression?
20. Given are the points $A = (1,2), B = (2,2), C = (2,1), D = (-1,4), E = (-2,-1), F = (-1,-1)$. Starting from initial clusters $C_1 = \{A\}$ which contains only the point A and $C_2 = \{D\}$ which contains only the point D, run the K-means algorithm and report the final clusters. Use L_1 distance as the distance between points which is given by $d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$ also draw the points on a 2D grid and mark the clusters.
21. Explain Q-Learning in detail.

(2 × 5 = 10 weightage)

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D 14109 (Pages : 3) Name..................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2021

(CBCSS)

Mathematics with Data Science

MTD 3C 14—SAMPLING THEORY AND DESIGN AND ANALYSIS OF EXPERIMENTS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Differentiate between SRSWR and SRSWOR with an example.
2. Give any two examples of probability sampling techniques.
3. What do you mean by systematic sampling ?
4. Explain the disadvantages of CRD.
5. Define local control.
6. In two-way ANOVA with m rows and n columns, the error degrees of freedom is.
7. What is meant by design of experiments ?
8. Define ANCOVA.

(8 × 1 = 8 weightage)

Turn over

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Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. Describe optimum allocation and proportional allocation in stratified sampling. Assess their efficiencies.

10. Explain how will you estimate population proportion.

11. Explain stratified random sampling. Estimate the gain due to stratification over simple random sampling.

Unit II

12. What are the basic principles of design of experiments ? Explain clearly the purpose of these principles in design of experiments.

13. Distinguish between CRD and RBD.

14. Explain the term "experimental error" and stating suitable examples.

Unit III

15. What is the motivation of the 3^k factorial designs ?

16. Discuss about multiple range tests.

17. Describing a 3^2 factorial experiment and explain how the Yate's technique is used to calculate sum of squares in the analysis.

(6 × 2 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. a) Discuss the problem of stratification. Obtain the optimum allocation for a fixed total sample size under stratified sampling.

    b) State the merits and demerits of optimum allocation to proportional allocation.

19. a) Derive the variance of systematic sampling interms of intraclass correlation coefficient.

    b) Write short notes on :

       i) Linear systematic sampling ; and

       ii) Circular systematic sampling.

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20. a) What does the term interaction refers to in experimental designs ?

b) Explain the model for one way analysis of variance stating its assumptions. Also discuss the role of fixed effects and mixed effects and the changes occur in each of the case.

21. a) Explain factorial design and obtain the main effects and interaction effects for a 2^2 factorial design and obtain its ANOVA table.

b) Explain the difference between one-factor at a time, full factorial and fractional factorial experiments.

(2 × 5 = 10 weightage)

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D 14108 (Pages : 3) Name.....................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2021

(CBCSS)

Mathematics with Data Science

MTD 3C 13—FUNCTIONAL ANALYSIS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section/Part shall remain the same.*
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4. *There will be an overall ceiling for each Section/Part that is equivalent to the maximum weightage of the Section/Part.*

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. State True *or* False, justify your claim : The space l^{∞} is separable.
2. State True *or* False, justify your claim : The space c_{00} is complete with respect to $\|\cdot\|_p$.
3. Give example for a discontinuous linear map.
4. Let X be a normed space over $\mathbb{K}$ and f be a non-zero linear functional on X. Show that f maps open sets into open sets.
5. Show that the space $P([a, b])$ of all polynomial functions defined on $[a, b]$ cannot become a Banach space under any norm.
6. Show that a proper dense subspace of a Banach space is not a Banach space.

Turn over

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7. State Uniform Boundedness Principle and show by examples that this need not hold in incomplete spaces.

8. Give example for a closed linear map, that is not continuous.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions from each of the following* **three** *units.*
Each question carries 2 *weightage.*

Unit I

9. Show that a linear functional f defined on a normed space is continuous if and only if the zero space $Z(f)$ is closed.

10. Show that a subspace of a Banach space is also a Banach space if and only if it is closed.

11. Show that if every linear map on a normed space is continuous, then the space is finite dimensional.

Unit II

12. State Hahn-Banach extension theorem for a normed space X. Let Y be a subspace of X. Prove that $x \in \overline{Y}$ if and only if $x \in X$ and $f(x) = 0$ whenever $f \in X'$ and $f|y = 0$.

13. Show that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

14. Show that the dual X' of a normed space X is always a Banach space.

Unit III

15. If X is a normed space and $P : X \to X$ is a projection, then show that P is closed if and only if the range $R(P)$ and the zero space $Z(P)$ are closed.

16. Give examples to show that the closed graph theorem and open mapping theorem need not hold in an incomplete space.

17. Let X be a normed space and $E \subset X$. Show that E is bounded if and only if $f(E)$ is bounded for every $f \in X'$.

(6 × 2 = 12 weightage)

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Section C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. Show that if every closed and bounded subset of a normed space X is compact, then X is finite dimensional.
19. State and prove Hahn-Banach separation theorem.
20. Show that if a linear map defined between two normed spaces is open, then it is surjective.
21. State and prove the two-norm theorem.

(2 × 5 = 10 weightage)

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D 14107 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2021

(CBCSS)

Mathematics with Data Science

MTD 3C 12—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend **all** questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

*Answer **all** questions.*
Each question carries 1 weightage.

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{z^n}{n^k}$ where $k \in \mathbb{N} \cup \{0\}$.

2. Sketch the image of the unit circle under the map $w = \frac{z}{1-z}$.

3. Let $\gamma : [a,b] \to \mathbb{R}$ be non-decreasing. Show that γ is of bounded variation.

4. Calculate the integral $\int_{\gamma} \frac{2z+1}{z^2+z+1} dz$ where γ is the circle $|z| = 2$.

5. Find the coefficients of z^m for $m \le 5$ in the power series expansion of $\frac{1}{\cos z}$.

Turn over

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6. Expand the function $\frac{2z+3}{z+1}$ as Power series of $\sum_k a_k (z-1)^k$, and find the radius of convergence.

7. Check whether the function $f(z) = \frac{\sin z}{z}$ has an essential singularity at $z = 0$.

8. Find the integral $\int_{|z+1|=1} \frac{z^2}{(z+1)^2 (z-1)} dz$.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. Show that $\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$.

10. Suppose γ is piecewise smooth and $f : [a,b] \to \mathbb{C}$ is continuous. Show that $\int_a^b f \, d\gamma = \int_a^b f(t) \gamma'(t) \, dt$.

11. Show that $R = \lim_{x \to \infty} \frac{|a_n|}{|a_{n+1}|}$ is the radius of convergence of the power series $\sum_n a_n (z-a)^n$ if the limit exists.

Unit II

12. Obtain Cauchy's estimate.

13. Establish first version of Cauchy's integral formula.

14. Show that every complex valued analytic function defined in a simply connected region has a primitive.

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Unit III

15. State and prove Argument principle.

16. Suppose f is analytic on the open unit disk D with $|f(z)| \leq 1$ for $z \in D$, and $f(0) = 0$. Prove that $|f'(0)| \leq 1$ and $|f(z)| \leq |z|$ for all $z \in D$.

17. Show that $a > 1$, $\int_0^{\pi} \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$.

(6 × 2 = 12 weightage)

Section C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. Suppose γ_0 and γ_1 are two closed rectifiable homotopic curves in G. For every analytic function $f \in G$ show that $\int_{\gamma_0}^{\pi} f = \int_{\gamma_1} f$.

19. State and prove maximum modulus theorem.

20. Show that every piecewise smooth curve $\gamma : [a, b] \to \mathbb{C}$ is of bounded variation and $V(\gamma) = \int_a^b | \gamma'(t) | dt$.

21. Show that $\int_0^{\infty} \frac{\log x}{1 + x^2} dx = 0$.

(2 × 5 = 10 weightage)

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D 14106 (Pages : 3) Name.....................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2021

(CBCSS)

Mathematics with Data Science

MTD 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2020 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Given $u(t)$ is a differentiable function of t, such that $u \cdot \frac{du}{dt} = 0$ for all t. Show that $\| u \|$ is constant.

2. Does a function $f : \mathbb{R}^n \to \mathbb{R}^m$ with continuous partial derivatives $\frac{\partial f}{\partial x_i}$ for $i = 1 \ldots n$, throughout an open region $R \subset \mathbb{R}^n$ shall be continuous on R ?

3. State the mean curvature and Gaussian curvature at a point p on a surface S in terms of the principal curvatures κ_1, κ_2 at p.

4. Define the normal curvature and the geodesic curvature for a unit-speed curve γ on an oriented surface S.

5. Let $\mathbf{u} = u_1\hat{\mathbf{i}} + u_2\hat{\mathbf{j}} + u_3\hat{\mathbf{k}}$ be a unit vector in $\mathbb{R}^3$. Compute the arc-length parametrization for the curve $\mathbf{r}(t) = (a_1 + tu_1)\hat{\mathbf{i}} + (a_2 + tu_2)\hat{\mathbf{j}} + (a_3 + tu_3)\hat{\mathbf{k}}$, where a_1, a_2, a_3 are constants in $\mathbb{R}$.

Turn over

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6. Define the second fundamental form for a surface patch σ.

7. Define the Gauss map on a surface S.

8. Describe what is an umbilic point on a surface S.

(8 × 1 = 8 weightage)

Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. State the inverse function theorem.

10. Suppose f maps a convex open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$, f is differentiable in E, and there is a real number M such that $\| f'(x) \| \le M$ for every $x \in E$. Show that $| f(b) - f(a) | \le M | b - a |$, for all $b, a \in E$.

11. Show that a linear operator A on $\mathbb{R}^n$ is invertible if and only if $\det(A) \neq 0$.

Unit II

12. Find the unit directions in which the function $f(x, y, z) = xe^y + z^2$ increase and decrease most rapidly at $P = \left(1, \ln 2, \frac{1}{2}\right)$.

13. Find the unit tangent and unit normal vectors for the circular motion $\mathbf{r}(t) = (\cos 2t)\hat{\mathbf{i}} + (\sin 2t)\hat{\mathbf{j}}$.

14. Show that the signed curvature of a circle with center at the origin and of radius R is $\frac{1}{R}$.

Unit III

15. Compute the second fundamental form for the surface of revolution :

$\sigma(u, v) = (f(u)\sin v, f(u)\cos v, g(u))$.

16. Calculate the Gauss map $\mathcal{G}$ for the paraboloid S given by the equation $z = x^2 + y^2$.

17. Derive the first fundamental form for the plane $\sigma(u, v) = \mathbf{a} + u\mathbf{p} + v\mathbf{q}$.

(6 × 2 = 12 weightage)

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Section C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. Let $U \to \mathbb{R}^3$ be a surface patch. Let $(u_0, v_0) \in U$ and $\delta > 0$ be such that the closed disc

$R_\delta = \{(u,v) \in \mathbb{R}^2 : (u-u_0)^2 + (v-v_0)^2 \le \delta^2\}$ with center at (u_0, v_0) and radius δ is contained in U.

Then show that $\lim_{\delta \to 0} \frac{|A_N(R_\delta)|}{|A_\sigma(R_\delta)|} = [K]$, where K is the Gaussian curvature of σ at (u_0, v_0).

19. Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. Show that f is continuously differentiable in E if and only if the partial derivatives $D_j f_i$, exist and are continuous on E for all $1 \le i \le m, 1 \le j \le n$.

20. Let S be a subset of $\mathbb{R}^3$ with the following property : for each point $p \in S$, there is an open subset W of $\mathbb{R}^3$ containing p and a smooth function $f : W \to \mathbb{R}$ such that :

    (a) $S \cap W = \{(x,y,z) \in W \mid f(x,y,z) = 0\}$, and

    (b) The gradient $\nabla f = (f_x, f_y, f_z)$ of f does not vanish at p.

    Then show that S is a smooth surface.

21. Show that the Möbius band is not orientable.

(2 × 5 = 10 weightage)

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